Scalar Feynman rule recap for matrix elements (after Q1):

$i\mathcal{M}(i\to f) = \Sigma$ all possible connected amputated Feynman diagrams (= drawings) in momentum space

Note: only use diagram components belonging to the given theory!

Fields encountered so far:

$$\hat{\phi}_I(x) = \int \frac{d\vec{p}}{(2\pi)^3\sqrt{2\omega_{\vec{p}}}}\left(\hat{a}_{\vec{p}}e^{-ip.x} + \hat{a}^\dagger_{\vec{p}}e^{+ip.x}\right)\Big|_{p^0=\omega_{\vec{p}}}$$

particle = antiparticle (mass m)

$$\hat{\psi}_I(x) = \int \frac{d\vec{p}}{(2\pi)^3\sqrt{2\omega_{\vec{p}}}}\left(\hat{b}_{\vec{p}}e^{-ip.x} + \hat{c}^\dagger_{\vec{p}}e^{+ip.x}\right)\Big|_{p^0=\omega_{\vec{p}}}$$

particle ≠ antiparticle (mass M)

$$\hat{\psi}^\dagger_I(x) = \int \frac{d\vec{p}}{(2\pi)^3\sqrt{2\omega_{\vec{p}}}}\left(\hat{b}^\dagger_{\vec{p}}e^{+ip.x} + \hat{c}_{\vec{p}}e^{-ip.x}\right)\Big|_{p^0=\omega_{\vec{p}}}$$

Arrow convention: $\hat{\phi}_I(x) \to$ no need for an arrow

$\hat{\psi}_I(x) \to$ arrow flows into spacetime point x

$\hat{\psi}^\dagger_I(x) \to$ " " out of " " "

particles flow along the arrow, antiparticles against it

$\Rightarrow$ the arrows represent particle-number flow & conservation.

also called internal lines

Scalar propagators: $\langle 0|T(\hat{\phi}_I(x)\hat{\phi}_I(y))|0\rangle \equiv \overbracket{\hat{\phi}_I(x)\hat{\phi}_I(y)} =$  $= \frac{i}{p^2-m^2+i\epsilon}$

drawing rule

$\langle 0|T(\hat{\psi}_I(x)\hat{\psi}^\dagger_I(y))|0\rangle \equiv \overbracket{\hat{\psi}_I(x)\hat{\psi}^\dagger_I(y)} =$  $= \frac{i}{p^2-M^2+i\epsilon}$

Not allowed: $\langle 0|T(\hat{\psi}_I(x)\hat{\psi}_I(y))|0\rangle = \langle 0|T(\hat{\psi}^\dagger_I(x)\hat{\psi}^\dagger_I(y))|0\rangle = 0$

$\Rightarrow$ only continuous arrows allowed!

External lines: a line that connects the initial or final state to a free field in the expansion of $i\hat{T}$ is called an external line

$\|i\rangle$	drawing	rule	$\langle f\|$	drawing	rule
incoming particle :		, 1	outgoing particle :		, 1
incoming antiparticle :		, 1	outgoing antiparticle :		, 1

we will come back to this!

⇒ antiparticles require explicit momentum bookkeeping, since they go against the arrow ! All momenta are on-shell !

Vertices: each interaction (involving more than two fields) that occurs in the expansion of $\hat{S} = T(e^{-i\int d^4x\, \boxed{\mathcal{H}_{int\,I}}(x)})$, with $\mathcal{H}_{int} = -\mathcal{L}_{int}$, is called a vertex and is indicated by a dot (i.e. the dots connected by the propagators / connected to the external lines).

P.&S. : $\hat{\mathcal{H}}_I$!

↳ **Feynman rules**:
- depend on the theory considered (see examples),
- multiply interaction strength in $\mathcal{L}_{int}$ by i,
- multiply by factors $n!$ for n identical fields in $\mathcal{L}_{int}$,
- energy and momentum are conserved at each vertex.

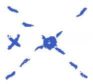
Loops: some internal (propagator) momenta are not fixed by energy-momentum conservation, these are called loop momenta

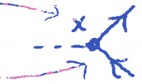
⇒ Feynman rule: $\int \frac{d^4 l}{(2\pi)^4}$ (l = loop momentum)

$\uparrow$ integral to be performed

Symmetry factor: to avoid double counting, each diagram should be divided by the so-called symmetry factor (= number of ways in which diagram components can be interchanged such that exactly the same diagram is obtained).

↳ this only occurs for interactions that involve identical fields!

Vertex examples: $\hat{\mathcal{L}}_{int_I}(x) = -\frac{\lambda}{4!}\hat{\phi}^4_I(x)$ ⇒  $= -i\frac{\lambda}{4!} * 4! = -i\lambda$,

$\hat{\mathcal{L}}_{int_I}(x) = -g\,\hat{\psi}^\dagger_I(x)\,\hat{\psi}_I(x)\,\hat{\phi}_I(x)$ ⇒  $= -ig * 1 = -ig$,

↳ continuous arrow !

derivative couplings $\partial_\mu \hat{\psi}_I(x)$ or $\partial_\mu \hat{\psi}^\dagger_I(x)$ in $\hat{\mathcal{L}}_{int_I}$

$\hat{c}^\dagger_{\vec q}\, e^{+iq\cdot x}$

$\partial_\mu \hat{\psi}_I(x)$:  $= -ip_\mu$

$\hat{b}_{\vec q}\, e^{-iq\cdot x}$

antiparticle out

particle in

$\hat{c}_{\vec q}\, e^{-iq\cdot x}$

$\partial_\mu \hat{\psi}^\dagger_I(x)$:  $= +ip_\mu$

$\hat{b}^\dagger_{\vec q}\, e^{+iq\cdot x}$

antiparticle in

particle out

} also here antiparticles require explicit momentum bookkeeping !

Q2 Feynman rule update for matrix elements:

Fields : generic notation for relevant fields and adjoint fields

$$\hat{F}_I(x) = \int \frac{d\vec{p}}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_\lambda \left(w(p,\lambda)\,\hat{a}^\lambda_{\vec{p}}\,e^{-ip.x} + w_c(p,\lambda)\,\hat{b}^{\lambda\dagger}_{\vec{p}}\,e^{+ip.x} \right)\Big|_{p^0=E_{\vec{p}}}$$

$$\hat{\bar{F}}_I(x) = \int \frac{d\vec{p}}{(2\pi)^3\sqrt{2E_{\vec{p}}}} \sum_\lambda \left(\bar{w}_c(p,\lambda)\,\hat{b}^\lambda_{\vec{p}}\,e^{-ip.x} + \bar{w}(p,\lambda)\,\hat{a}^{\lambda\dagger}_{\vec{p}}\,e^{+ip.x} \right)\Big|_{p^0=E_{\vec{p}}}$$

with λ referring to polarization d.o.f., $\hat{a}$ and $\hat{a}^\dagger$ to particle modes, $\hat{b}$ and $\hat{b}^\dagger$ to antiparticle modes, and $w, w_c, \bar{w}, \bar{w}_c$ to polarization vectors (spanning spin space)

↳ scalars : $w = w_c = \bar{w} = \bar{w}_c = 1$; Dirac fermions : $w(p,\lambda) = u^\lambda(p)$, $w_c(p,\lambda) = v^\lambda(p)$, $\bar{w}(p,\lambda) = \bar{u}^\lambda(p)$, $\bar{w}_c(p,\lambda) = \bar{v}^\lambda(p)$; gauge bosons : $w = \bar{w}_c = \epsilon^\lambda_\mu(p)$, $\bar{w} = w_c = \epsilon^{\lambda *}_\mu(p)$

Propagator update : drawing  , rule $\dfrac{i}{p^2 - m^2 + i\epsilon} \sum_\lambda w(p,\lambda)\,\bar{w}(p,\lambda)$

with

$$\sum_\lambda w(p,\lambda)\,\bar{w}(p,\lambda) = \begin{cases} 1 & \text{(scalars)} \\ \not{p} + m \leftarrow \boxed{\text{matrix in spinor space}} & \text{(Dirac fermions)} \\ -g_{\mu\nu} + p_\mu p_\nu / m^2 & (m \neq 0 \text{ gauge bosons}) \\ -g_{\mu\nu} + \dfrac{p_\mu n_\nu + p_\nu n_\mu}{n.p} - \dfrac{p_\mu p_\nu}{(n.p)^2} & (m = 0 \text{ gauge bosons}) \end{cases}$$

Generalized external lines (see FR1) :

		drawing	rule
$\lvert i\rangle$	incoming particle :		$\sqrt{Z_F}\; w(p,\lambda)$
	incoming antiparticle :		$\sqrt{Z_F}\; \bar{w}_c(p,\lambda)$
$\lvert f\rangle$	outgoing particle :		$\sqrt{Z_F}\; \bar{w}(p,\lambda)$
	outgoing antiparticle :		$\sqrt{Z_F}\; w_c(p,\lambda)$

enters @ loop level

with $p^0 = E_{\vec{p}}$ (on-shell) and $\sqrt{Z_F}$ = wave-function renormalization factor depending on the type of particle

Arrow convention revisited : additional rule for fermions → on top of indicating the particle-number flow, the arrow also tells you to insert γ-matrices and spinors while going against the arrow !

Fermionic minus signs :

- *) diagrams with two identical fermions interchanged differ by a minus sign (due to Fermi statistics) ;
- *) each closed loop of fermion propagators (fermion loop) receives a minus sign and involves a trace in spinor space.

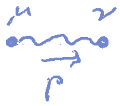
↑ no open Dirac indices

Feynman rules specific for QED :

solid line —— for fermions (e.g. e^-),
wiggly line ∼ for photons (γ)

matrix in spinor space

QED vertex :  $= -iq\gamma^\mu \leftrightarrow \mathcal{L}^{int}_{QED_I} = -\mathcal{H}^{int}_{QED_I} = -q\bar{\psi}_I\gamma^\mu\psi_I A_{\mu_I}$ for Dirac fermions with charge q,

Photon prop :  $= \frac{-i}{p^2+i\epsilon}\left(g_{\mu\nu} - \frac{p_\mu n_\nu + p_\nu n_\mu}{n\cdot p} + \frac{p_\mu p_\nu}{(n.p)^2}\right) \xrightarrow{\text{effectively}} \frac{-ig_{\mu\nu}}{p^2+i\epsilon}$.

consequence of gauge invariance, photon couples to conserved currents