Scalar Feynman rule recap for matrix elements (after Q1):

$i\mathcal{M}(i \to f) = \sum$ all possible connected amputated Feynman diagrams (= drawings) in momentum space

Note: only use diagram components belonging to the given theory!

Fields encountered so far: $\hat{\phi}_I(x) = \int \frac{d\vec{p}}{(2\pi)^3 \sqrt{2\omega_{\vec{p}}}} \left(\hat{a}_{\vec{p}}\, e^{-ip.x} + \hat{a}^{\dagger}_{\vec{p}}\, e^{+ip.x} \right) \Big|_{p^0 = \omega_{\vec{p}}}$

particle = antiparticle (mass m)

$$\hat{\varphi}_I(x) = \int \frac{d\vec{p}}{(2\pi)^3 \sqrt{2\omega_{\vec{p}}}} \left(\hat{b}_{\vec{p}}\, e^{-ip.x} + \hat{c}^{\dagger}_{\vec{p}}\, e^{+ip.x} \right) \Big|_{p^0 = \omega_{\vec{p}}}$$

particle ≠ antiparticle (mass M)

$$\hat{\varphi}^{\dagger}_I(x) = \int \frac{d\vec{p}}{(2\pi)^3 \sqrt{2\omega_{\vec{p}}}} \left(\hat{b}^{\dagger}_{\vec{p}}\, e^{+ip.x} + \hat{c}_{\vec{p}}\, e^{-ip.x} \right) \Big|_{p^0 = \omega_{\vec{p}}}$$

Arrow convention: $\hat{\phi}_I(x) \to$ no need for an arrow

$\hat{\varphi}_I(x) \to$ arrow flows into spacetime point x

$\hat{\varphi}^{\dagger}_I(x) \to$ " " out of " " "

particles flow along the arrow, antiparticles against it

$\Rightarrow$ the arrows represent particle-number flow & conservation.

(also called internal lines)

Scalar propagators: $\langle 0| T(\hat{\phi}_I(x)\hat{\phi}_I(y))|0\rangle \equiv \hat{\phi}_I(x)\hat{\phi}_I(y) = y \overset{p}{\cdots\cdots} x = \frac{i}{p^2 - m^2 + i\epsilon}$

$\langle 0| T(\hat{\varphi}_I(x)\hat{\varphi}^{\dagger}_I(y))|0\rangle \equiv \hat{\varphi}_I(x)\hat{\varphi}^{\dagger}_I(y) = y \xrightarrow{p} x = \frac{i}{p^2 - M^2 + i\epsilon}$

(drawing, rule)

Not allowed: $\langle 0| T(\hat{\varphi}_I(x)\hat{\varphi}_I(y))|0\rangle = \langle 0| T(\hat{\varphi}^{\dagger}_I(x)\hat{\varphi}^{\dagger}_I(y))|0\rangle = 0$

$y \bullet\!\leftarrow\!\rightarrow\!\bullet\, x$ $\qquad$ $y \bullet\!\rightarrow\!\leftarrow\!\bullet\, x$

$\Rightarrow$ only continuous arrows allowed!

External lines: a line that connects the initial or final state to a free field in the expansion of $i\hat{T}$ is called an external line

$\|i\rangle$	drawing	rule	$\langle f\|$	drawing	rule
incoming particle :	$\overset{k}{\rightarrow}\bullet$	1	outgoing particle :	$\bullet\!\xrightarrow{p}$	1
incoming antiparticle :	$\overset{k'}{\leftarrow}\bullet$	1	outgoing antiparticle :	$\bullet\!\xleftarrow{p'}$	1

we will come back to this!

$\Rightarrow$ antiparticles require explicit momentum bookkeeping, since they go against the arrow ! All momenta are on-shell !

(P.&S. : $\hat{\mathcal{H}}_I$) !

Vertices : each interaction (involving more than two fields) that occurs in the expansion of $\hat{S} = T\left(e^{-i\int d^4x\, \hat{\mathcal{H}}_{int\,I}(x)}\right)$, with $\mathcal{H}_{int} = -\mathcal{L}_{int}$, is called a vertex and is indicated by a dot (i.e. the dots connected by the propagators / connected to the external lines).

Feynman rules :
- depend on the theory considered (see examples),
- multiply interaction strength in $\mathcal{L}_{int}$ by i,
- multiply by factors $n!$ for n identical fields in $\mathcal{L}_{int}$,
- energy and momentum are conserved at each vertex.

Loops : some internal (propagator) momenta are not fixed by energy-momentum conservation, these are called loop momenta

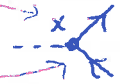
$\Rightarrow$ Feynman rule : $\int \frac{d^4 \ell}{(2\pi)^4}$ (ℓ = loop momentum)

integral to be performed

Symmetry factor : to avoid double counting, each diagram should be divided by the so-called symmetry factor (= number of ways in which diagram components can be interchanged such that exactly the same diagram is obtained).

this only occurs for interactions that involve identical fields !

Vertex examples : $\hat{\mathcal{L}}_{int_I}(x) = -\frac{\lambda}{4!}\hat{\phi}^4_I(x) \Rightarrow$  $= -i\frac{\lambda}{4!} * 4! = -i\lambda$,

$\hat{\mathcal{L}}_{int_I}(x) = -g\,\hat{\varphi}^\dagger_I(x)\,\hat{\varphi}_I(x)\,\hat{\phi}_I(x) \Rightarrow$  $= -ig * 1 = -ig$,

continuous arrow !

derivative couplings $\partial_\mu \hat{\varphi}_I(x)$ or $\partial_\mu \hat{\varphi}^\dagger_I(x)$ in $\hat{\mathcal{L}}_{int_I}$

$\partial_\mu \hat{\varphi}_I(x)$ ($\ni \hat{c}^\dagger_{\vec q}\, e^{+iq\cdot x}$, $\hat{b}_{\vec q}\, e^{-iq\cdot x}$) : antiparticle out  particle in $= -ip_\mu$

$\partial_\mu \hat{\varphi}^\dagger_I(x)$ ($\ni \hat{c}_{\vec q}\, e^{-iq\cdot x}$, $\hat{b}^\dagger_{\vec q}\, e^{+iq\cdot x}$) : antiparticle in  particle out $= +ip_\mu$

also here antiparticles require explicit momentum bookkeeping !