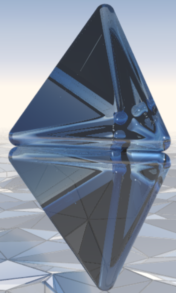
Effective Dynamics in non-perturbative quantum gravity

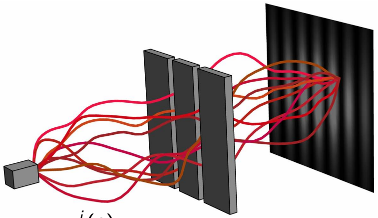
Timothy Budd



QUIST & Thesis talk, March 15, 2012
Institute for Theoretical Physics

Path integral approach to quantum gravity

- Quantum mechanics according to Feynman



$x^i(t)$

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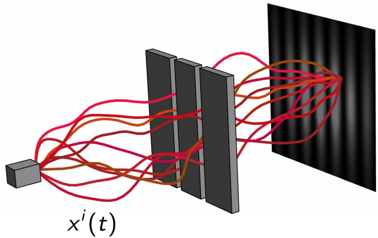

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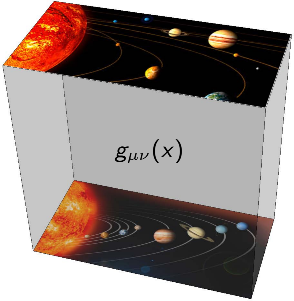

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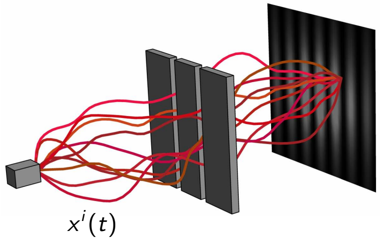

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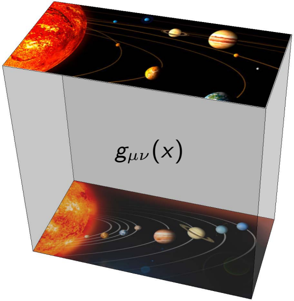

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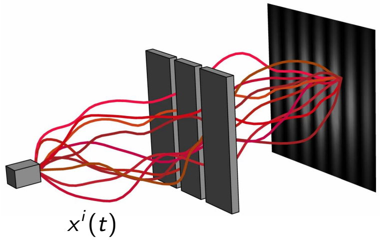

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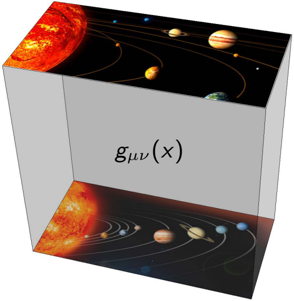

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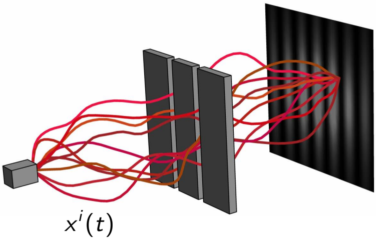

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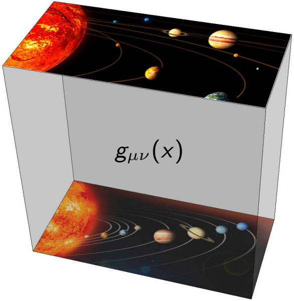

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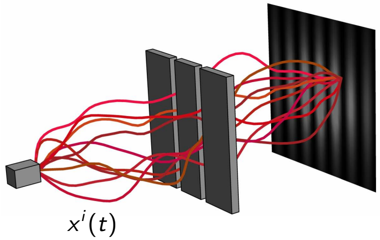

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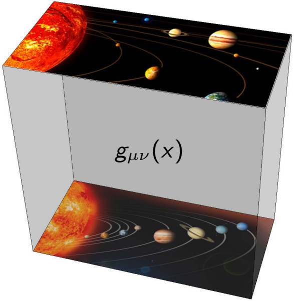


- Can we combine these pillars of modern theoretical physics?
- First try pure gravity: sum over space-time geometries.
- Hope: UV complete model gravity + reproduces general relativity.
- This talk: given a microscopic model (CDT), establish its effective dynamics.

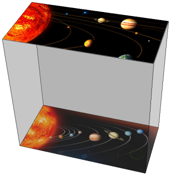
Outline

- Introduction to CDT in 2+1 dimensions
- Previous results for topology $S^2 \times \mathbb{R}$: spatial volume
- Effective action for CDT: conformal model problem
- Alternative to Einstein–Hilbert
- Moduli as observables for CDT with topology $T^2 \times \mathbb{R}$
- Monte Carlo measurements vs ansatz
- Summary and outlook

CDT in 2+1 dimensions

- Causal Dynamical Triangulation is a regularization of the Euclidean path integral over geometries

$$Z = \int \frac{\mathcal{D}g}{Diff} e^{-S_{EH}[g]} \quad \rightarrow \quad Z_{CDT} = \sum_{\text{triangulations}\, T} \frac{1}{C_T} e^{-S_{CDT}[T]}.$$



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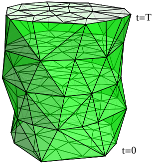

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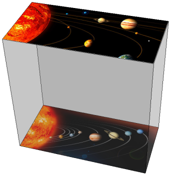

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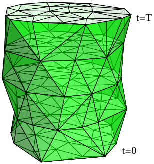

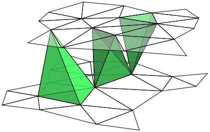

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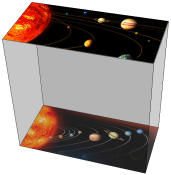

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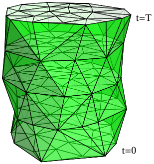

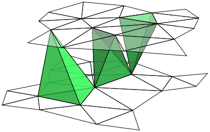

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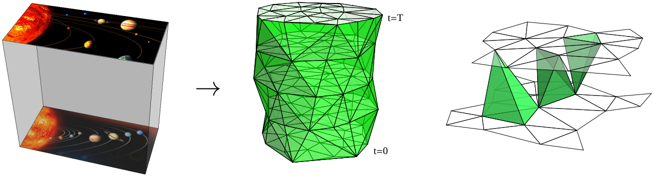

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- The Euclidean Einstein–Hilbert action $S[g] = -\kappa \int d^3x \sqrt{g}(R - 2\Lambda)$ evaluated on the piecewise linear geometry leads to

$$S_{CDT}[T] = k_3 N_3 - k_0 N_0.$$

Monte Carlo methods for CDT

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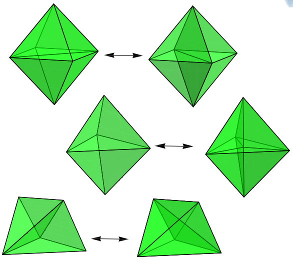


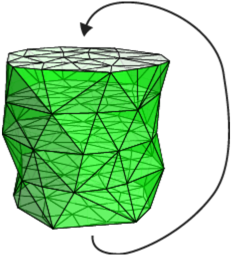
- We use Monte Carlo methods to approximate these:

$$\langle \mathcal{O} \rangle_{N_3} \approx \frac{1}{n} \sum_{i=1}^{n} \mathcal{O}(T_i),$$

where the $\{T_i\}$ is a large set of random triangulations generated by applying a large number of random update moves satisfying a detailed balance condition.

CDT with spherical spatial topology

- Take periodic time and spatial topology S^2



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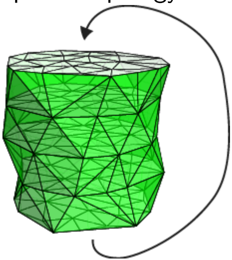
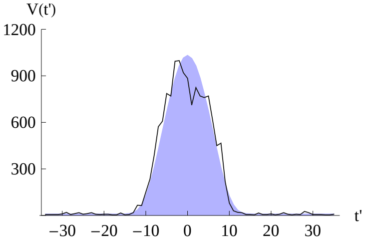


- $\langle V(t)\rangle \propto \cos^2(\sqrt{c_1}\,t)$ plus minimal "stalk"



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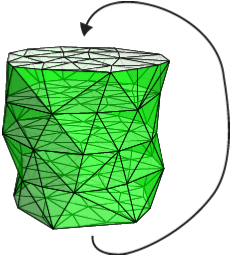

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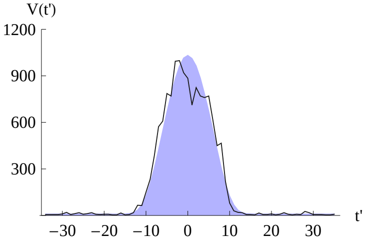

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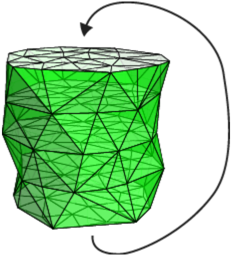

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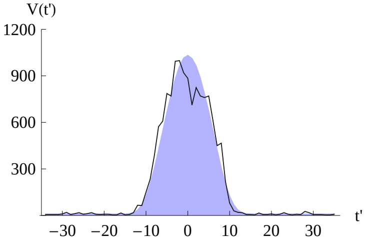

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- Also correlations

$$\langle V(t)V(t')\rangle - \langle V(t)\rangle\langle V(t')\rangle \propto \left(\left.\frac{\delta^2 S_{eff}}{\delta V(t)\delta V(t')}\right|_{\langle V(t)\rangle} \right)^{-1}$$

well-described by this action for suitable values $c_0, c_1 > 0$.

CDT with spherical spatial topology (continued)

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- Indefinite metric! Positive definite on traceless directions, negative definite on trace/conformal directions in superspace.

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generalizes naturally to

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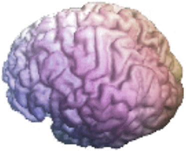
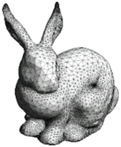
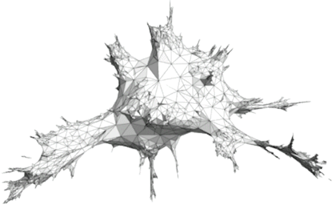
- We have ended up with an ansatz in the realm of Euclidean (projectable) Hořava–Lifshitz gravity.

New observables

- Need observable $f : \{\text{2d triangulations}\} \to \mathbb{R}$. Whole journals are dedicated to shape recognition in medical imaging, computer graphics, etc. However the random geometries in CDT are much wilder.

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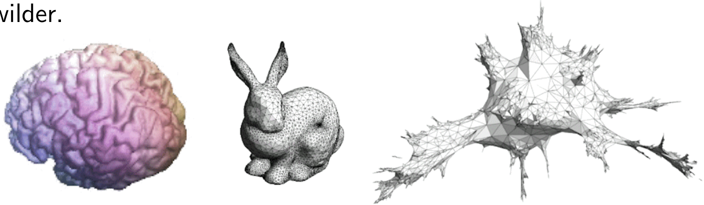
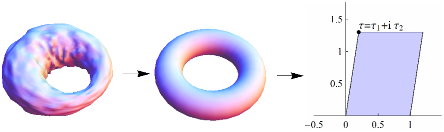


- The conformal structure of a torus! Moduli τ_i (Teichmüller parameters) .



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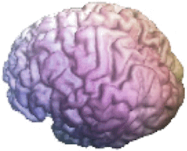
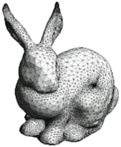
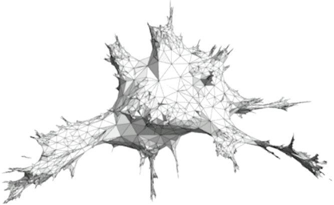

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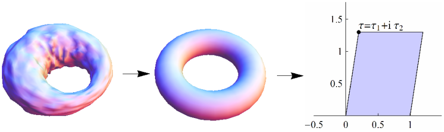
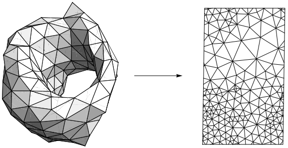
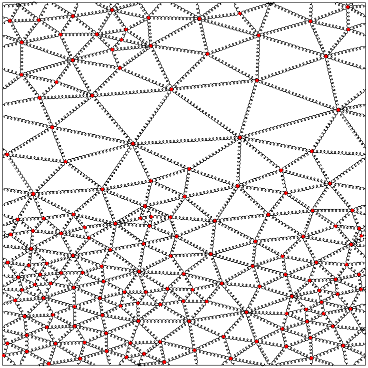




Moduli on a triangulation

- Replace every edge by a perfect spring: periodic discrete harmonic embedding.



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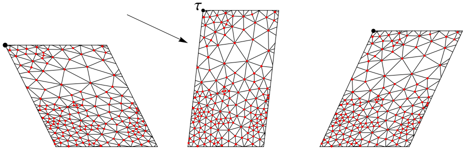

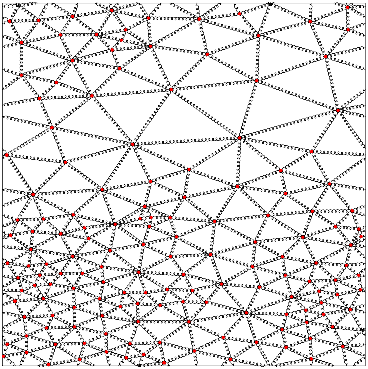

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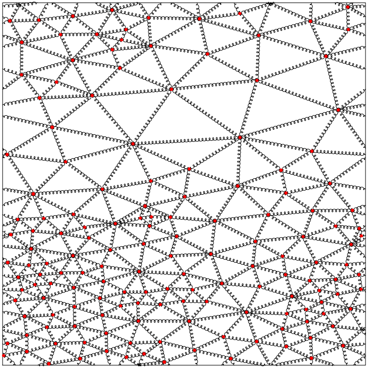
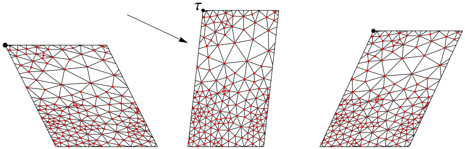
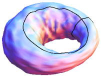
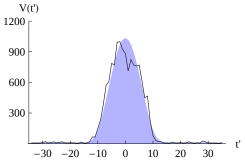


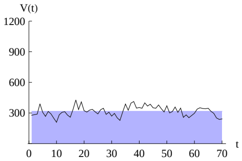


- The functions τ_i : $\Big\{$  $\Big\} \to \mathbb{R}$ are well-defined.

CDT with spatial torus topology

- Try periodic boundary conditions $T^2 \times S^1$. No breaking of time translation symmetry:





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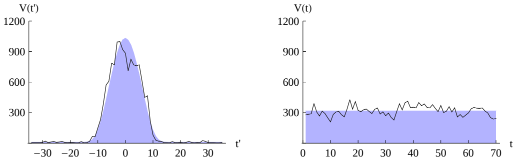

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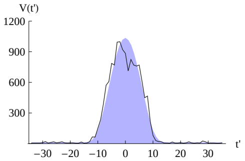

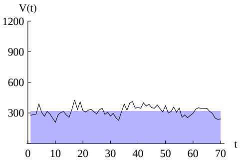

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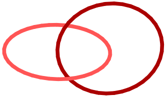

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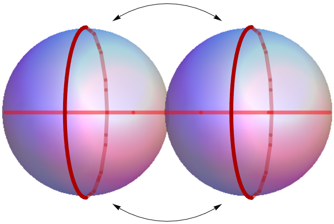

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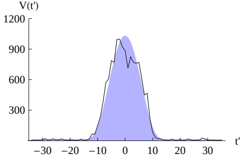

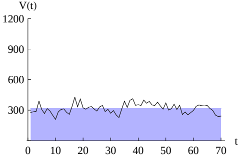

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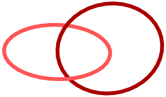

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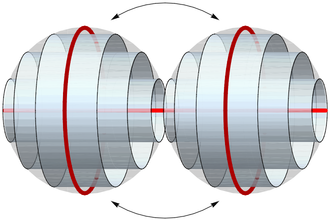

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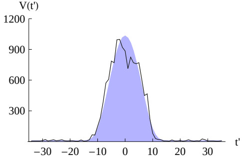

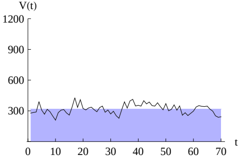

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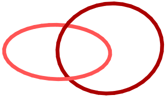

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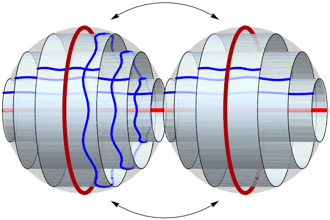

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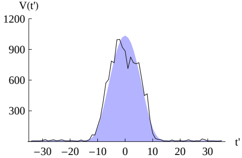

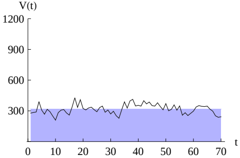

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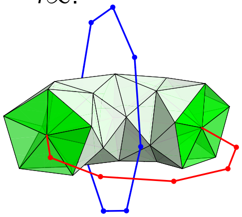

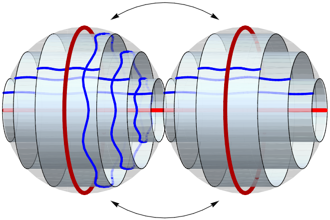

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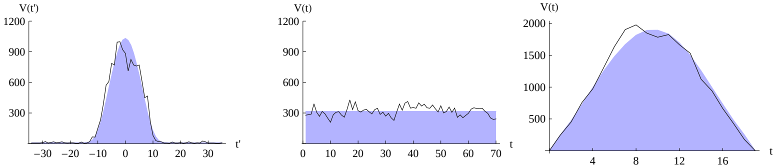

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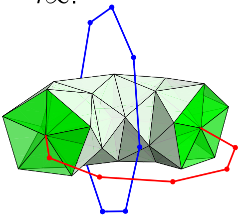

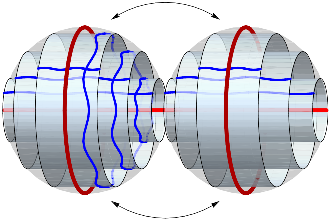

Comparison with ansatz

- Evaluating

$$S_{ansatz} = \kappa \int_0^T dt \int d^2x \sqrt{g}(\dot{g}_{ab}\mathcal{G}_\lambda^{abcd}\dot{g}_{cd} - U[g]),$$

on spatially flat cosmology $ds^2 = dt^2 + V(t)\hat{g}_{ab}(\tau)dx^a dx^b$, $\hat{g}_{ab}(\tau, x) = \frac{1}{\tau_2}\begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}$ gives

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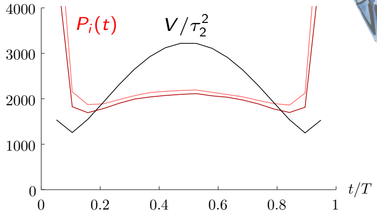
- Prefactors in kinetic term related to inverse of correlators:

$$(\langle \tau_i(t)\tau_j(t')\rangle - \langle \tau_i(t)\rangle\langle \tau_j(t')\rangle)^{-1} \propto \left(P_i(t)\frac{d^2}{dt^2} + \cdots\right)\delta(t-t')\delta_{ij}$$

- Compare $P_i(t)$ and $\langle V(t)/\tau_2(t)^2 \rangle$ in CDT simulation.



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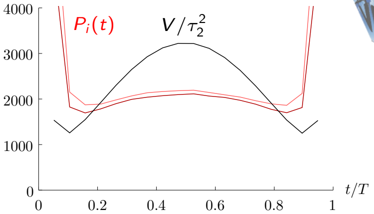

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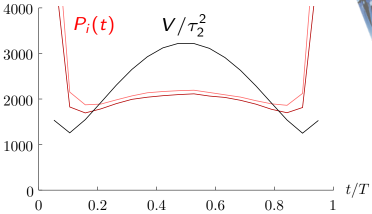

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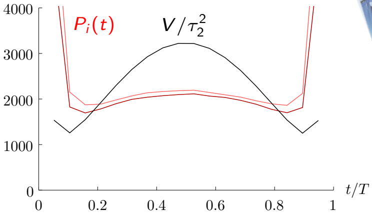

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- $A[g]$ is the expected change in τ under a random deformation δg_{ab} (normalized w.r.t. $\mathcal{G}$)

$$A[g] = \frac{\delta_{ij}}{4\tau_2^2} \int d^2x \sqrt{g} \frac{\delta \tau_i}{\delta g_{ab}} \mathcal{G}_{abcd} \frac{\delta \tau_j}{\delta g_{cd}}.$$

- $A[g]$ can be worked out explicitly for a general metric g_{ab} on the torus

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\begin{aligned}
A[g] &= \frac{\delta_{ij}}{4\tau_2^2} \int d^2x \sqrt{g}\, \frac{\delta \tau_i}{\delta g_{ab}} \mathcal{G}^{\lambda}_{abcd} \frac{\delta \tau_j}{\delta g_{cd}} \\
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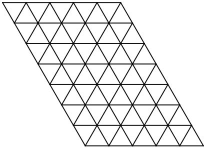
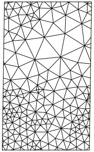
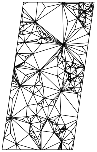
- A natural discretization of $A[g]$ to triangulations T is $A[T] = \sum_{\sigma\in T}\mathrm{area}(\sigma)^2$, where $\mathrm{area}(\sigma)$ is the area of the triangle σ in the harmonic embedding of T in the plane.



$VA[T] = 1$

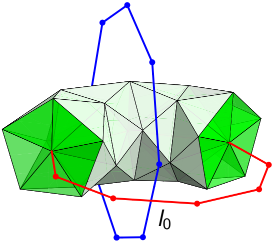


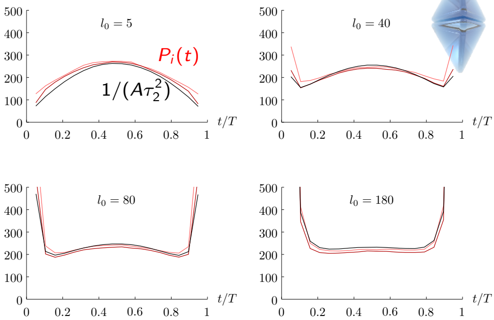
$VA[T] \approx 1.9$



$VA[T] \approx 4.9$

- Retry: compare P_i and $\langle 1/(A(t)\tau_2(t)^2)\rangle$.





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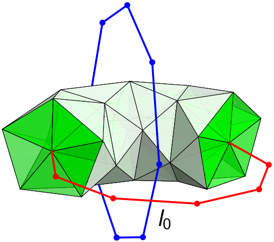

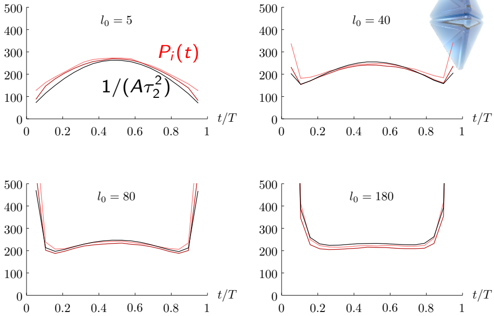

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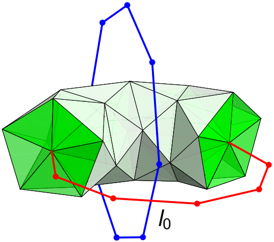

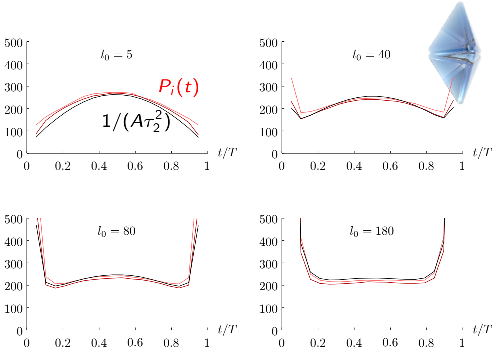

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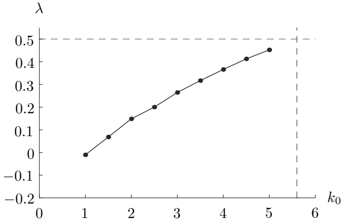
$$S[V,\tau] = \kappa \int dt \Big((\frac{1}{2} - \lambda) \frac{\dot{V}^2}{V} + \frac{1}{2A} \frac{\dot{\tau}_1^2 + \dot{\tau}_2^2}{\tau_2^2} - U(V,\tau) \Big).$$

- Similar analysis for spatial volume $V(t)$ leads to a measurement of $\kappa(\frac{1}{2} - \lambda)$. Combining both we get a value $\lambda \approx 0.22$ (for coupling $k_0 = 2.5$).

- We can perform this analysis for various couplings k_0 ($\approx$ bare Newton's constant).



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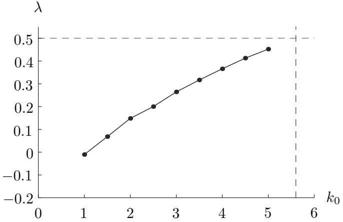
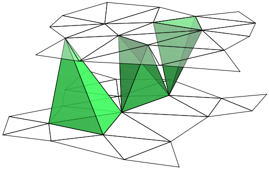
- Plot shows that as $k_0 \to k_0^*$ the spatial volumes $V(t), V(t+1)$ decouple, while the moduli $\tau_i(t), \tau_i(t+1)$ remain coupled!





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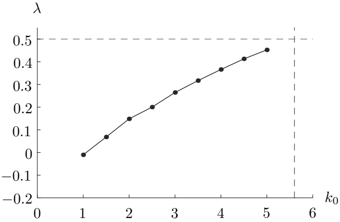

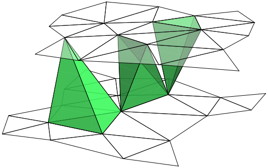

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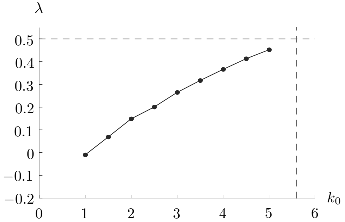
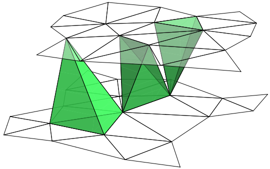
- However, seems to hold more generally for trace vs traceless d.o.f. in g_{ab}. See extrinsic curvature measurements in chapter 6 of my thesis.





Summary

- In order to describe the measurements of the moduli in CDT with topology $T^2 \times \mathbb{R}$ we have introduced a non-covariant ansatz for an effective action.
- The correlations in V and τ are compatible with a kinetic term given by a generalized Wheeler–DeWitt metric $\mathcal{G}_\lambda$ with $\lambda < 1/2$ (recall that $\lambda = 1$ in GR).
- This strengthens an earlier observed connection between CDT and Hořava–Lifshitz gravity, a potentially renormalizable non-covariant generalization of GR.

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Don't forget: Tuesday at 2:30 PM, my thesis defense!

Thanks!

Appendices

Measurement of τ in the continuum

- Any metric g_{ab} on the torus is conformally flat and up to diffeomorphisms the flat unit-volume metrics are given by
$$\hat{g}_{ab}(\tau, x) = \frac{1}{\tau_2} \begin{pmatrix} 1 & \tau_1 \\ \tau_1 & \tau_1^2 + \tau_2^2 \end{pmatrix}.$$

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$$\Delta = d\delta + \delta d \quad \text{(Hodge Laplacian)}$$

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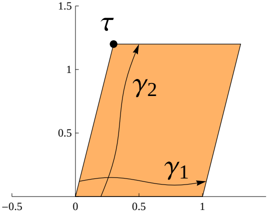

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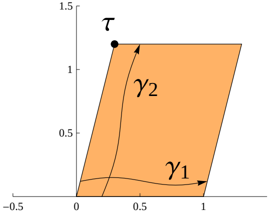

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Measurement of τ for torus triangulations

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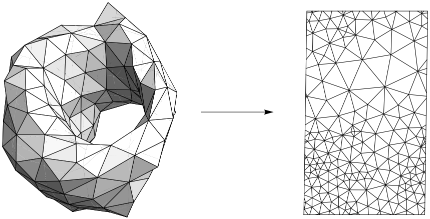
- Once we have these ingredients we can construct discrete conformal maps:



Discrete differential forms

- In 2d triangulations we have
  - Vertices: 0-simplices denoted by i,
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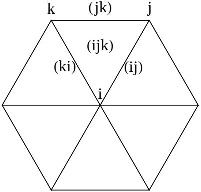

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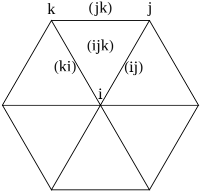

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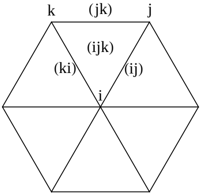

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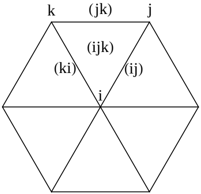

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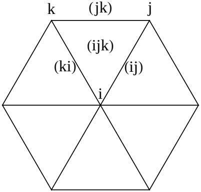

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