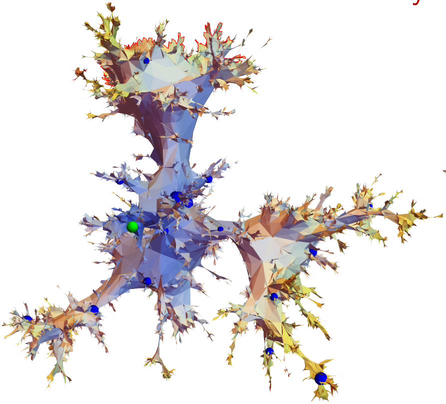
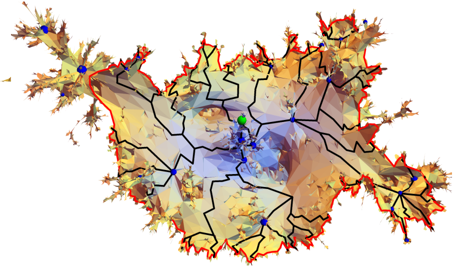
Quantum gravity in Paris, 21-03-2017

Escaping universality in two-dimensional quantum gravity

Timothy Budd





Based on joint work with Nicolas Curien, Cyril Marzouk.

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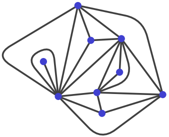
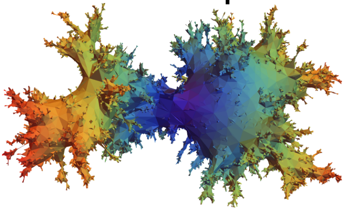
Brownian sphere



edge length $\sim N^{-1/4}$

$N \to \infty$



[Le Gall, Miermont, ...]

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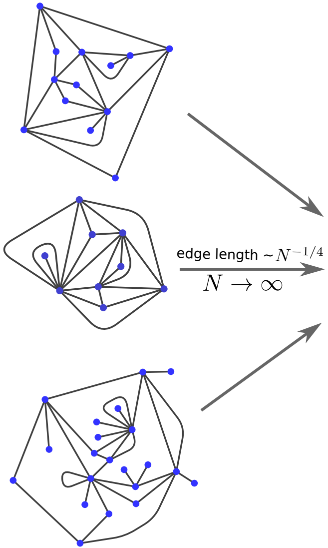

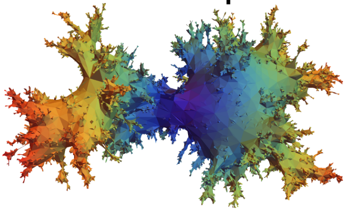

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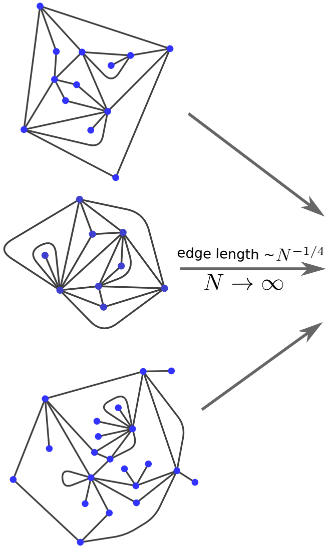
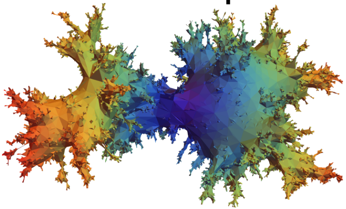




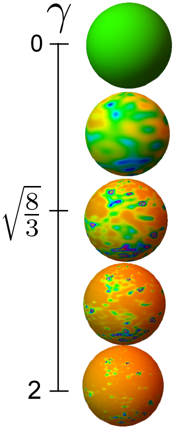
- It's a metric space, i.e. a set with a distance function $d(x, y)$
- Induced topology S^2 almost surely.
- Genuinely fractal: $d_H = 4$

[Le Gall, Miermont, ...]

Liouville Quantum Gravity

$$g_{ab} = e^{\gamma\phi}\hat{g}_{ab} \qquad Z = \int \mathcal{D}\phi\, e^{-S[\phi,\hat{g}]}$$

$$S[\phi,\hat{g}] = \int \mathrm{d}^2x\sqrt{\hat{g}}\left(\hat{g}^{ab}\partial_a\phi\partial_b\phi + \hat{R}\phi + \lambda e^{\gamma\phi}\right)$$



Planar maps

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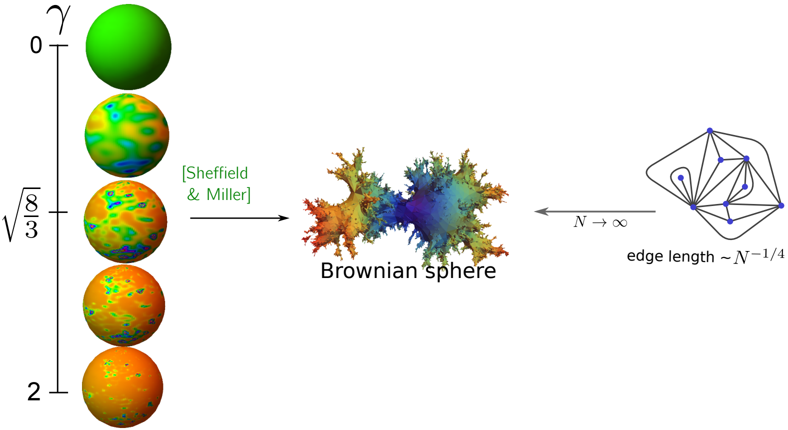

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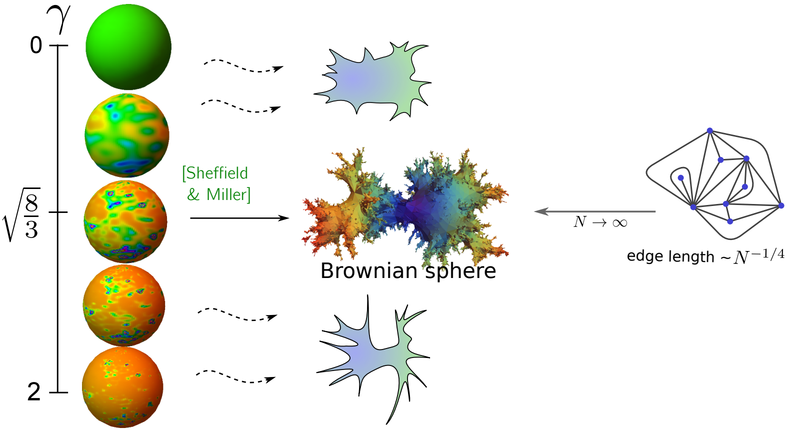

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Planar maps with "matter"

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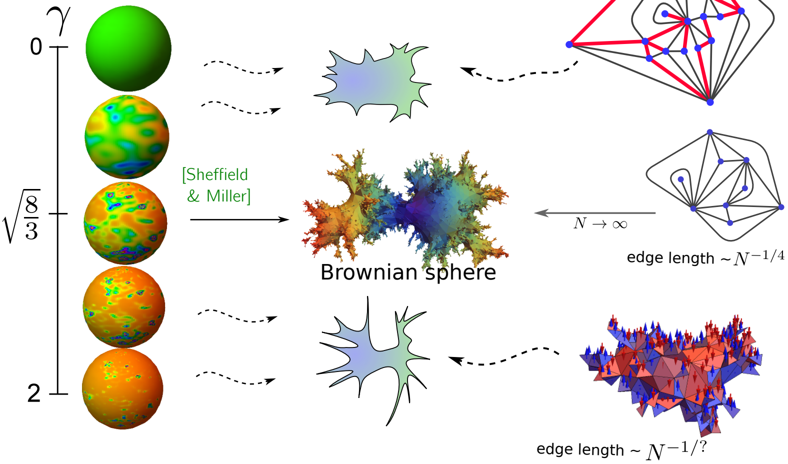

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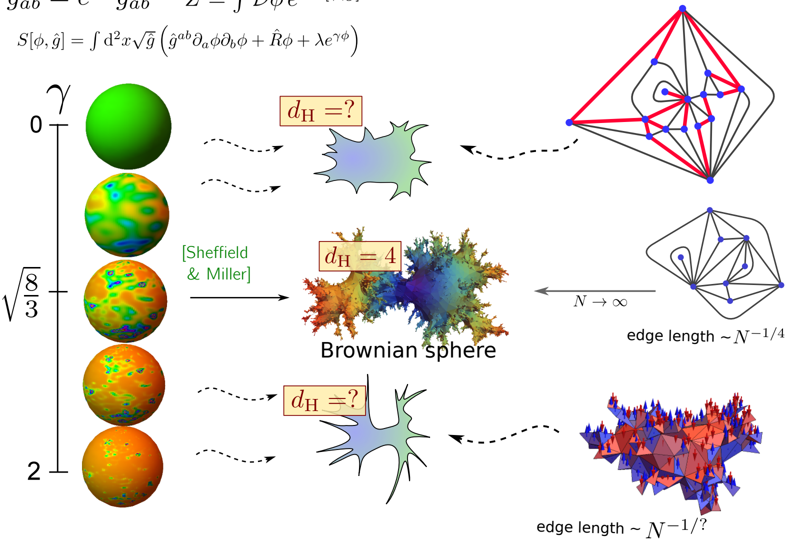
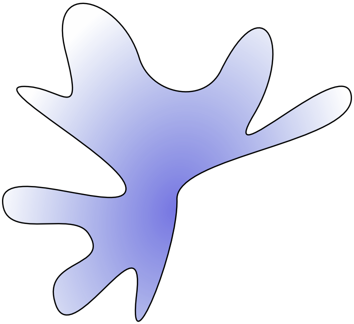


- Take any random metric space on S^2 (with no holes or atoms).



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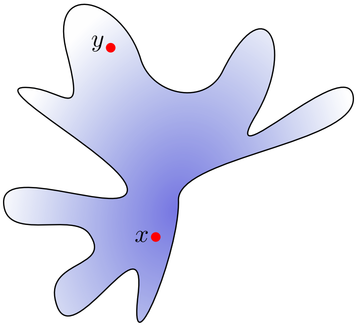

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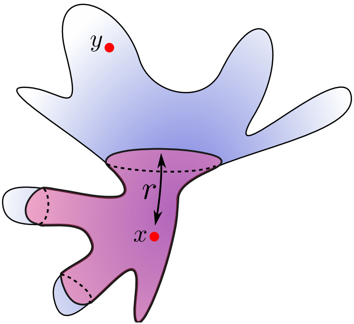

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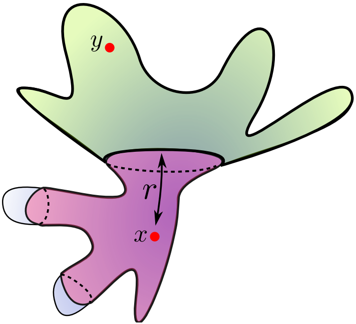

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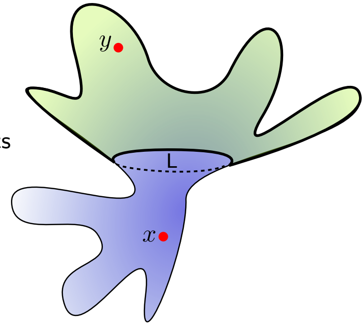

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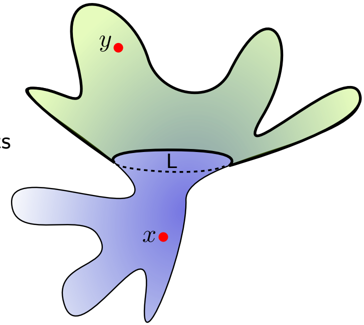

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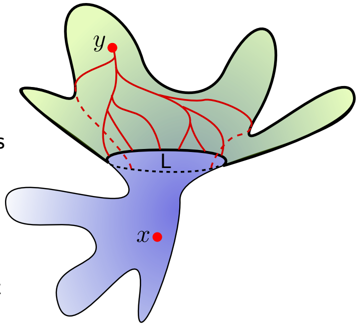

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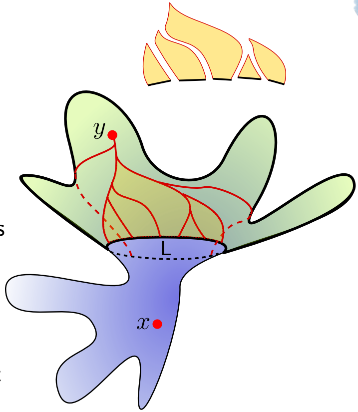

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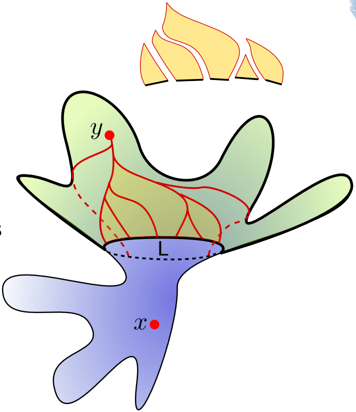

- Two ways to escape Brownian universality
  - violate (1) e.g. by matter coupling, but studying geometry hard
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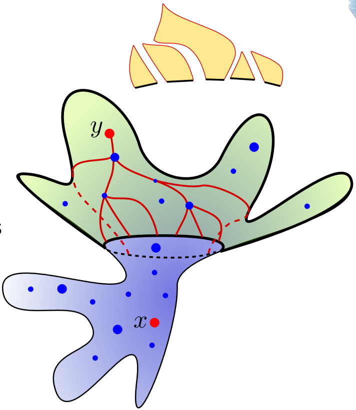

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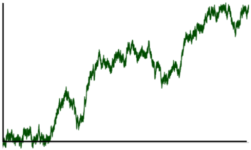
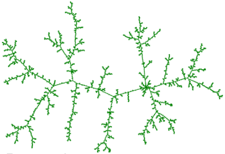
  - violate (1) e.g. by matter coupling, but studying geometry hard
  - keep (1)+(2) but violate (3) e.g. by encouraging geodesics to meet in special points ("with exceptionally large negative curvature")

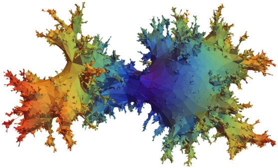
From Brownian to stable



Brownian motion



Brownian tree (Aldous' CRT)



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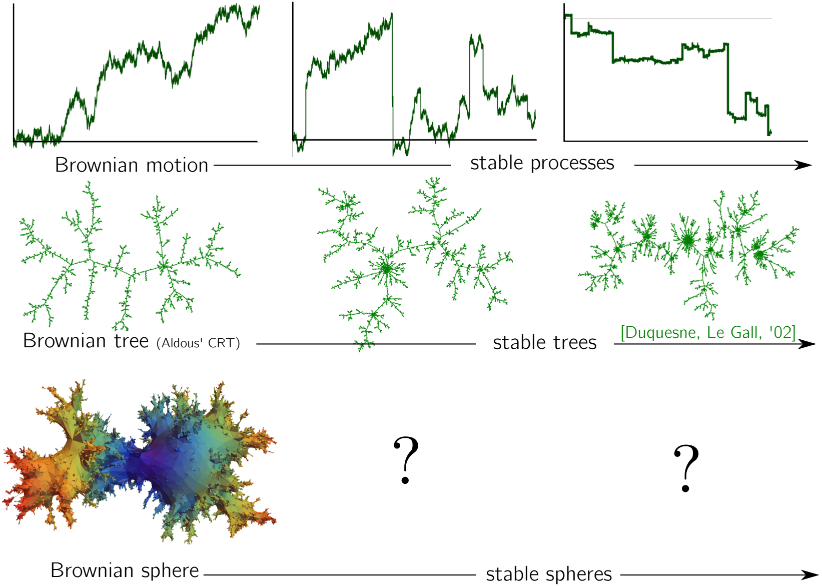
From Brownian to stable

Relax "continuity", retain self-similarity & Markov property



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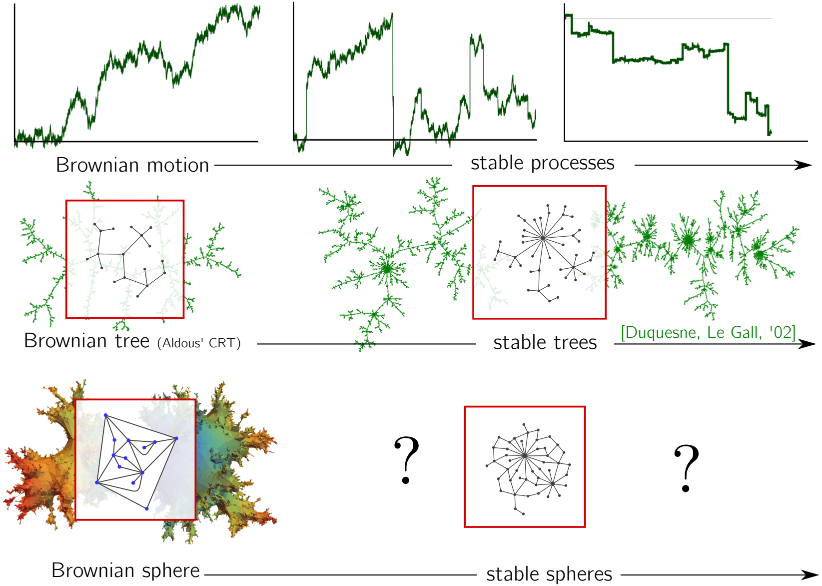
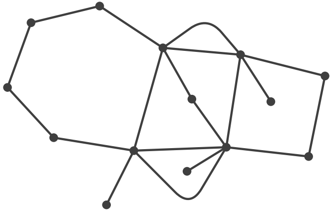


Boltzmann planar maps

- A planar map $\mathfrak{m}$ is a multigraph embedded in S^2 modulo deformation. In addition, rooted and bipartite.



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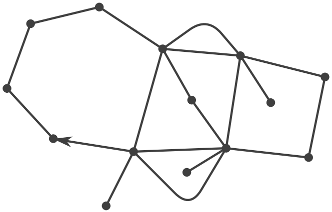

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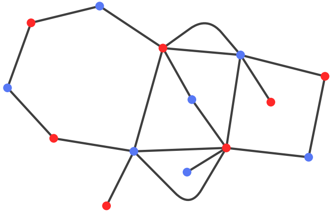

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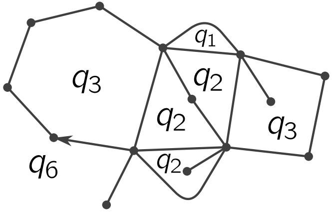

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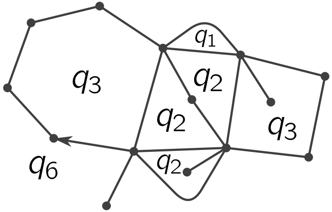

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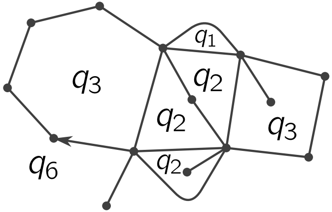

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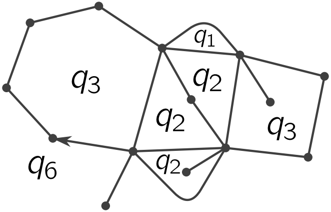

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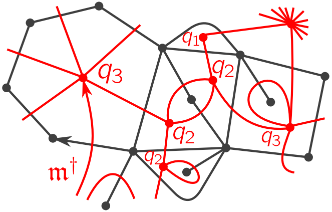
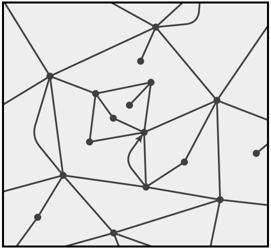
- The dual map $\mathfrak{m}^\dagger$ has vertices of high degree.



Infinite Boltzmann planar maps

- **q**-BPMs are small, so we first condition them to have a large fixed number of vertices N.
- As $N \to \infty$ there is a well-defined "local" limit, the Infinite BPM, in the sense that the law of the neighbourhood of the root converges. [Björnberg, Stefánsson, '14] [Stephenson, '14]



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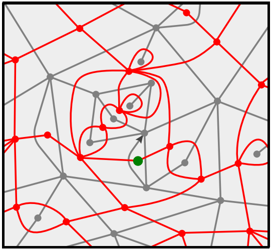

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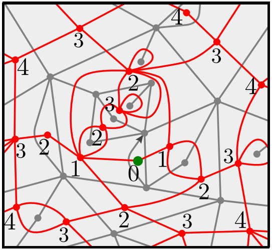

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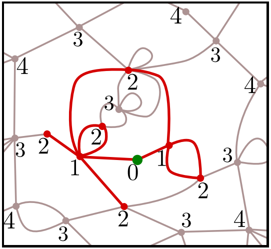

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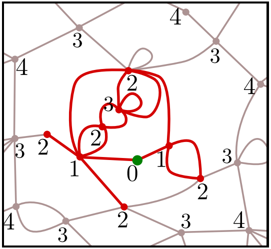

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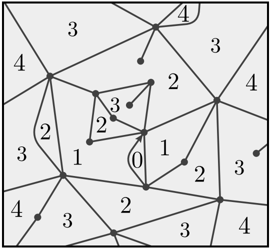

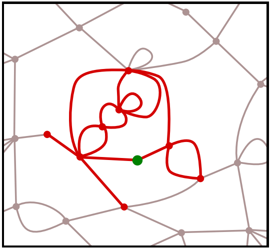

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Infinite Boltzmann planar maps

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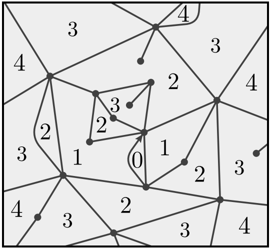

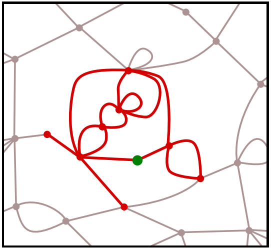

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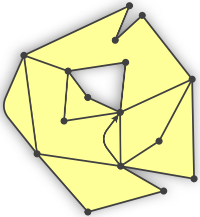

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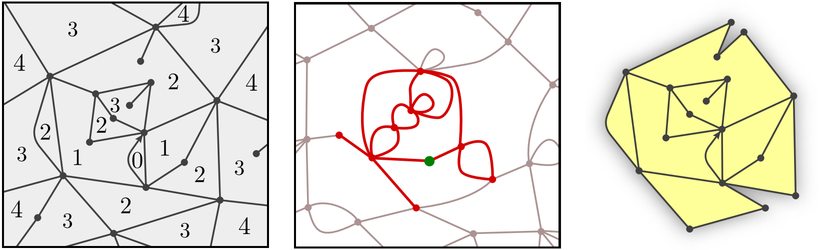
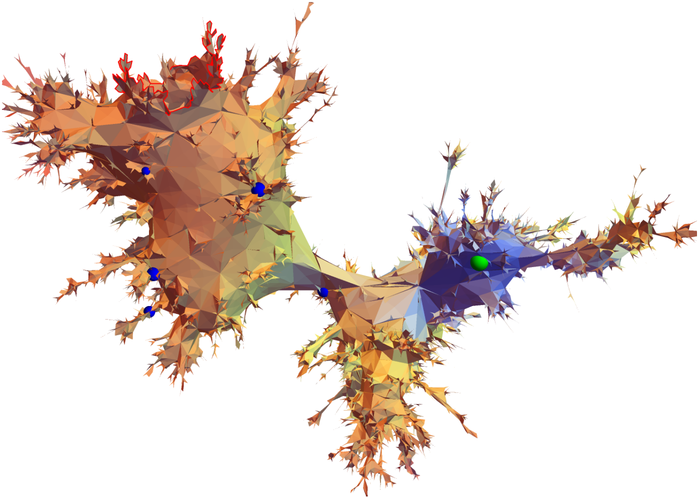


$r = 2$

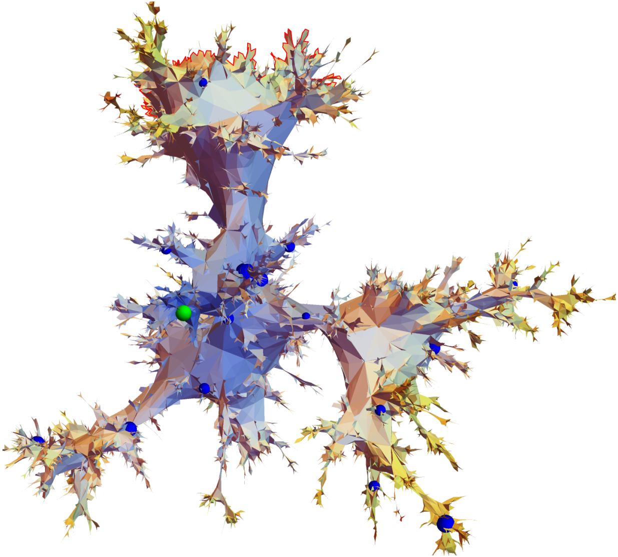
Simulations

$a = 2.45$



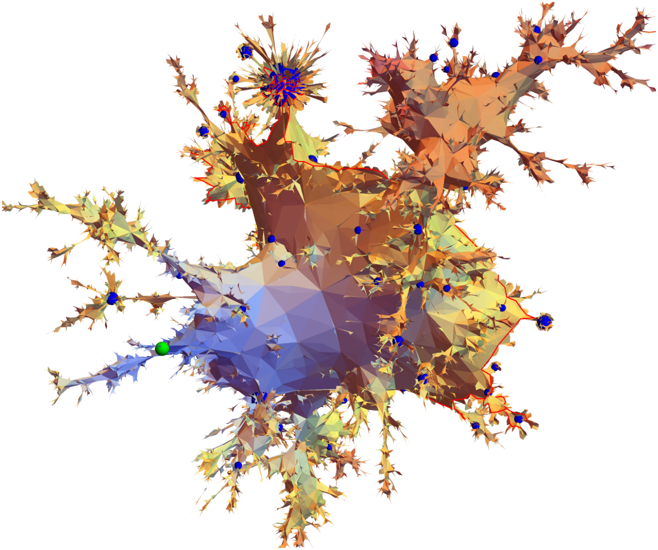
Simulations

$a = 2.35$



Simulations

$a = 2.3$



Simulations

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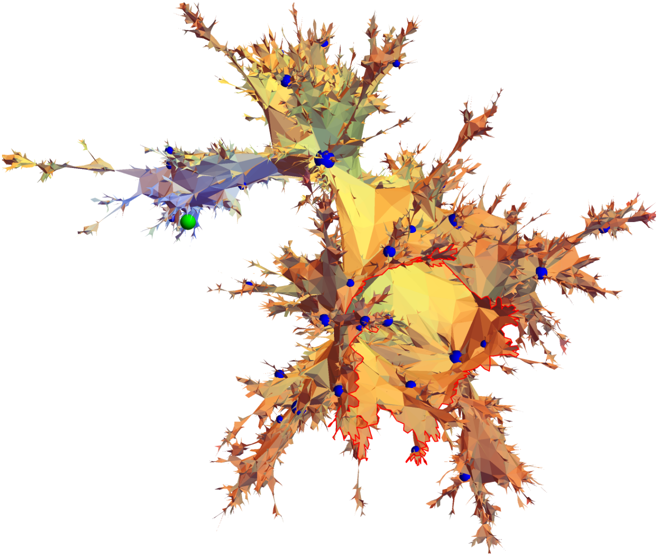
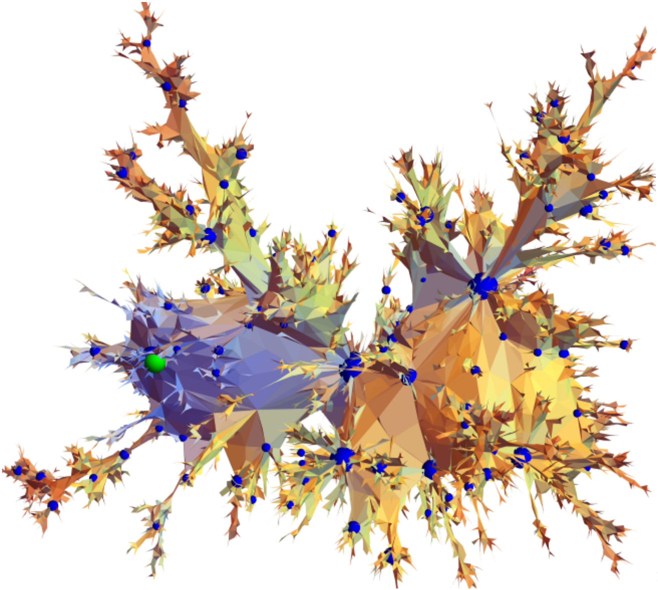


Simulations

$a = 2.0$



Simulations

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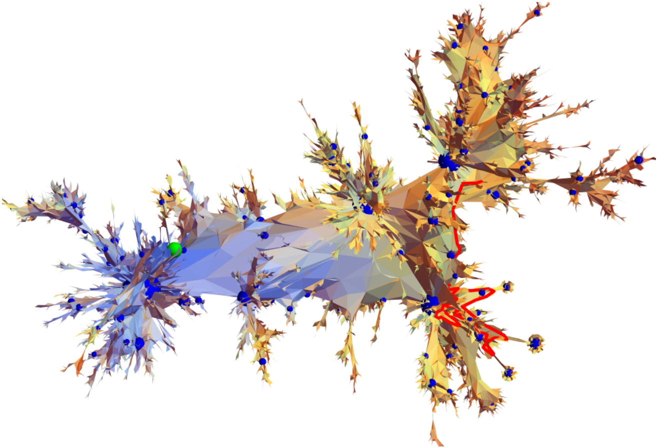
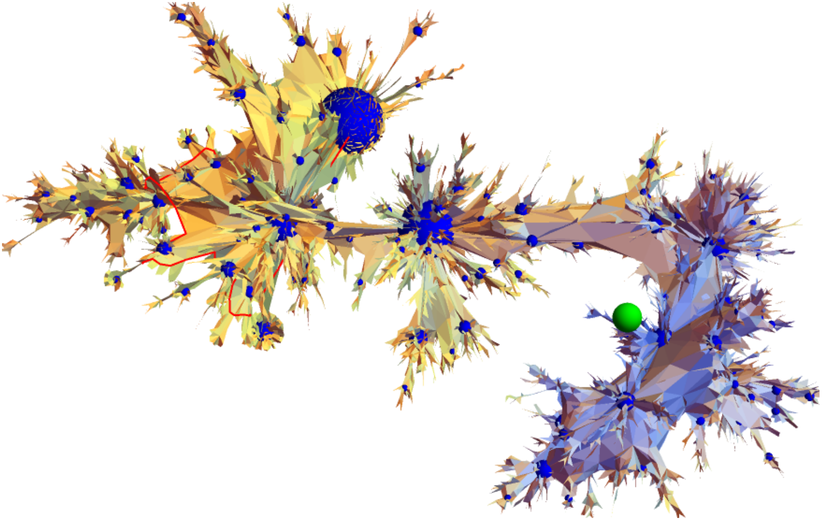


Simulations

$a = 1.8$



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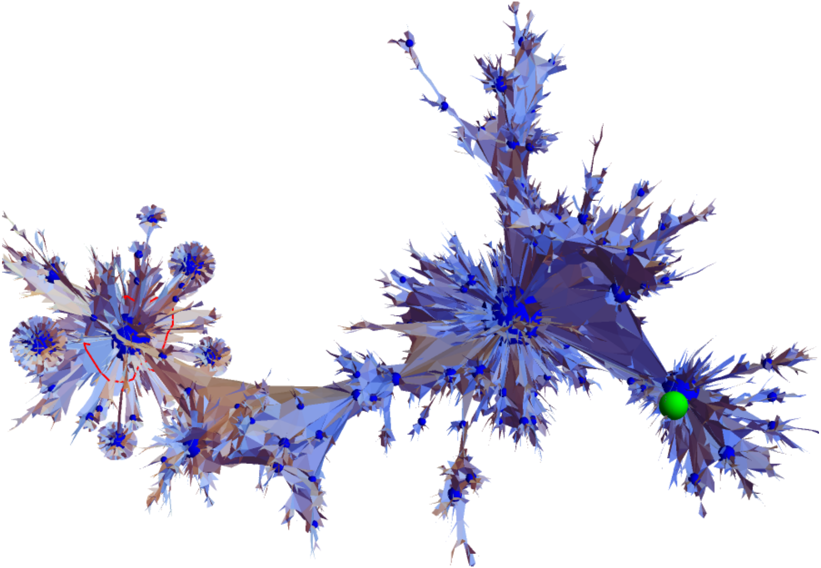
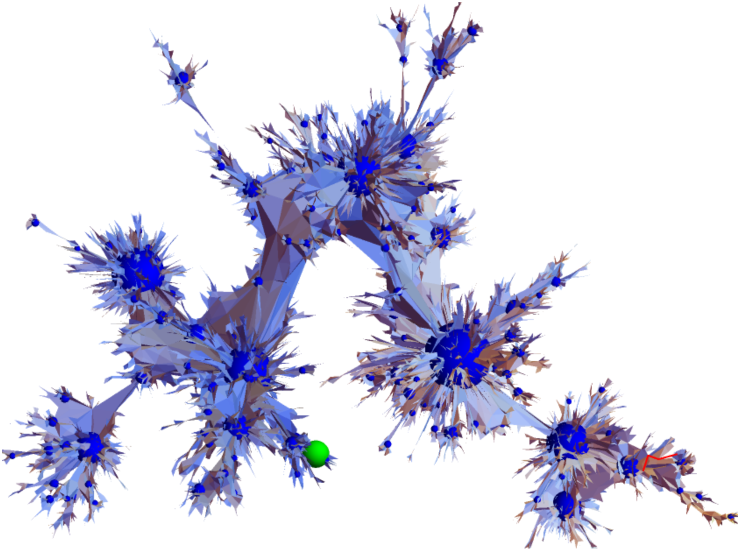
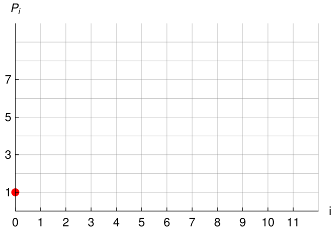


Simulations

$a = 1.7$



Peeling by layers of a **q**-IBPM



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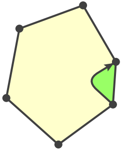

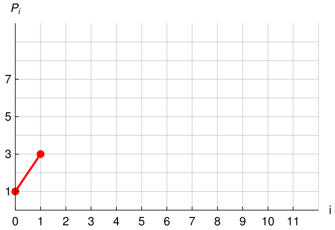

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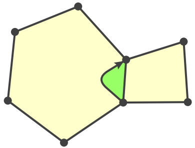

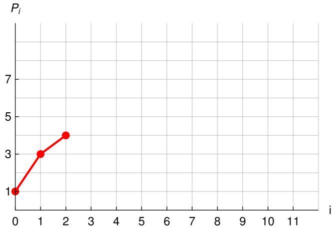

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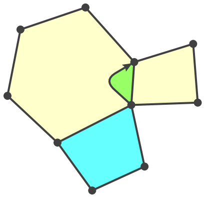

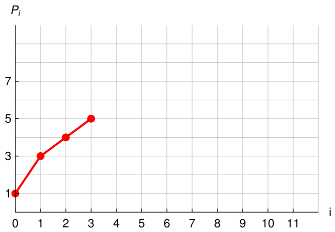

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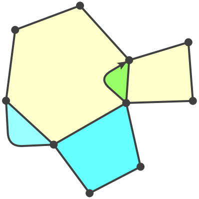

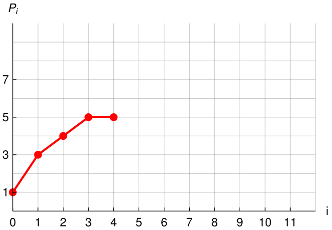

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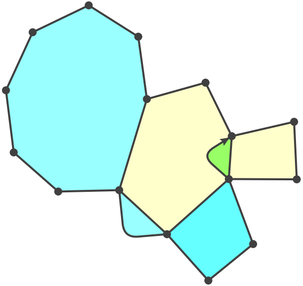

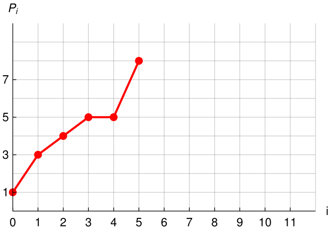

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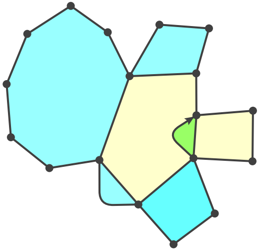

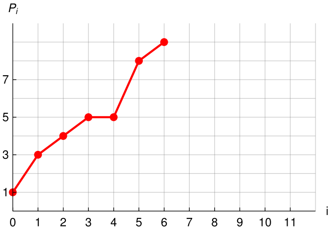

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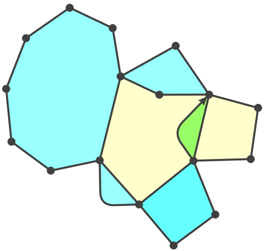

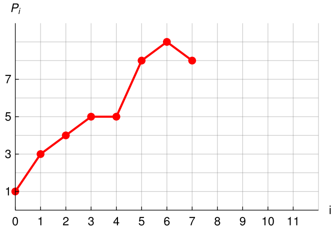

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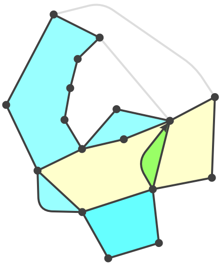

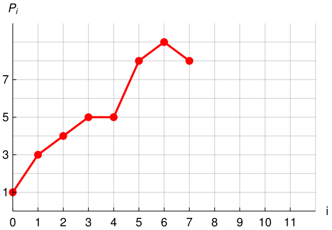

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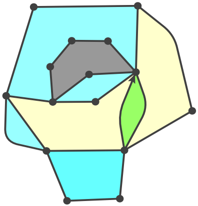

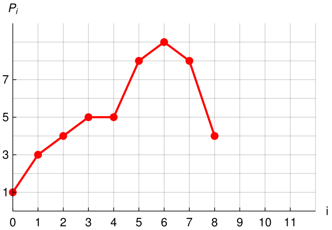

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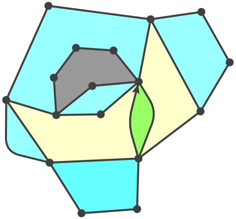

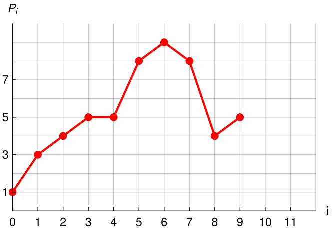

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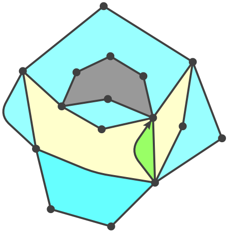

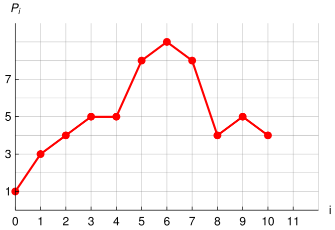

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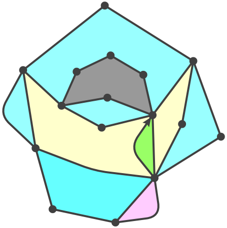

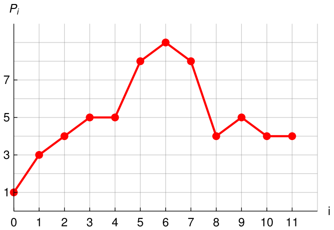

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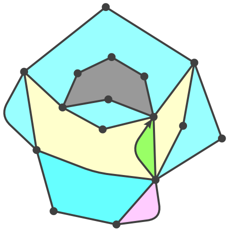

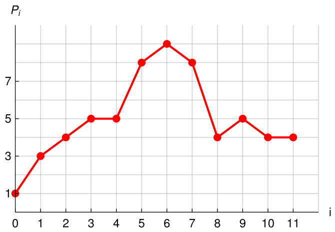

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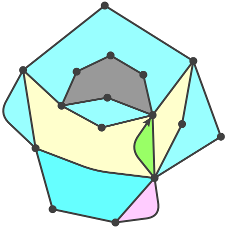

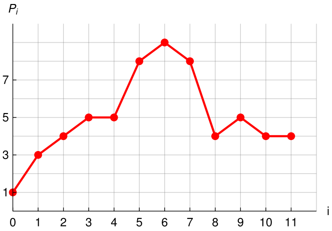

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$$\mathbb{P}(P_{i+1} = P_i + k) = \frac{h^{\uparrow}(P_i + k)}{h^{\uparrow}(P_i)}\nu_{\mathbf{q}}(k) \qquad h^{\uparrow}(l) = 2l \cdot 4^{-l}\binom{2l}{l}$$

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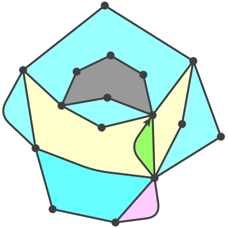

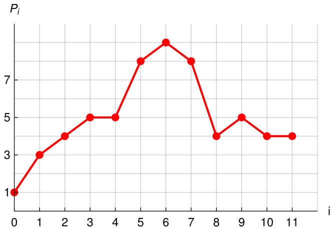

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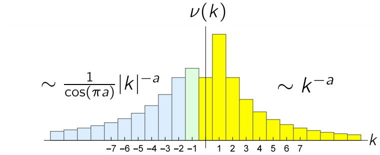
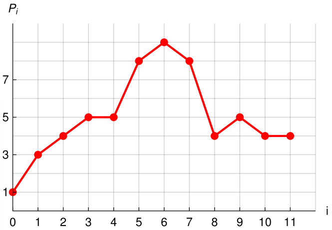
- In fact $\{\nu(k) : h^{\uparrow} \text{ does this job }\} \leftrightarrow \{\mathbf{q} \text{ critical }\}$. [TB,'15]

Peeling by layers of a q-IBPM

- When $\mathbf{q}_k \sim \mathrm{p} \cdot \mathrm{c}^{-k} \cdot k^{-a}$, $a \in (\frac{3}{2}, \frac{5}{2})$ then $\nu(k)$ is of form:





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Asymptotic growth [TB, Curien, '16] [TB, Curien, Marzouk, '17]

	$\frac{3}{2} < a < 2$	$a = 2$	$2 < a < \frac{5}{2}$
Perimeter after n steps	$\approx n^{\frac{1}{a-1}}$	$\approx n$	$\approx n^{\frac{1}{a-1}}$

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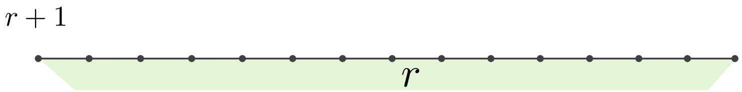

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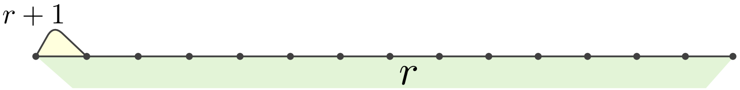

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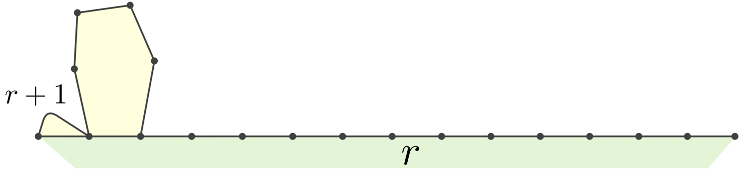

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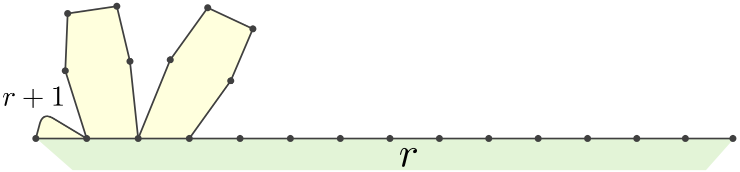

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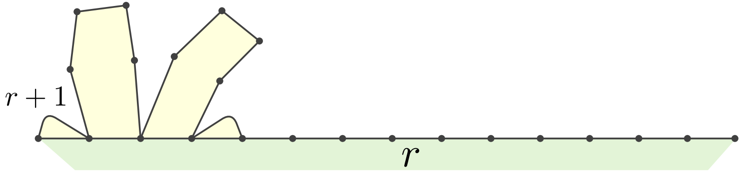

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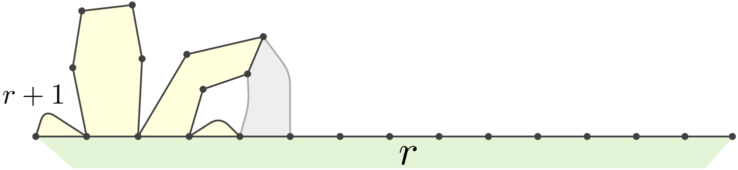

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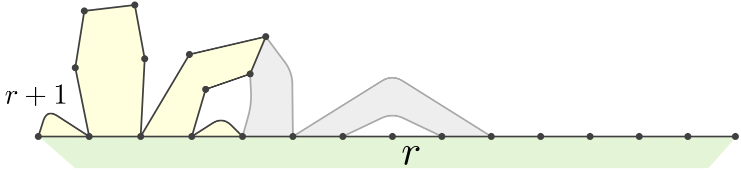

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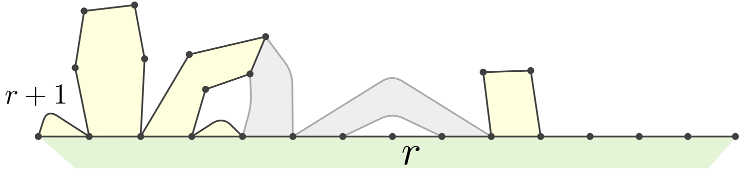

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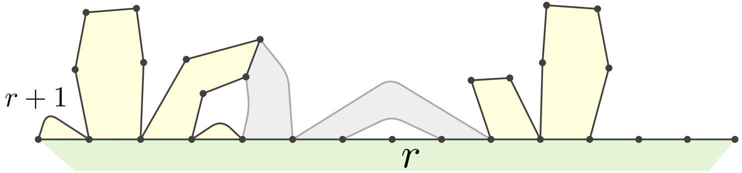
	$\frac{3}{2} < a < 2$	$a = 2$	$2 < a < \frac{5}{2}$
Perimeter after n steps	$\approx n^{\frac{1}{a-1}}$	$\approx n$	$\approx n^{\frac{1}{a-1}}$



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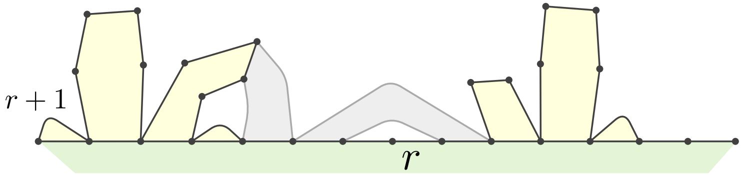

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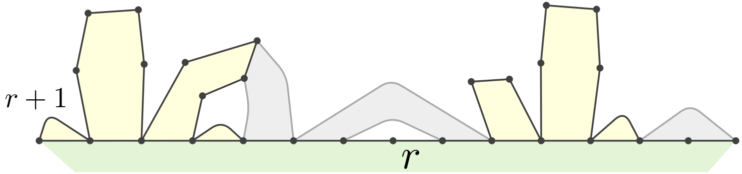

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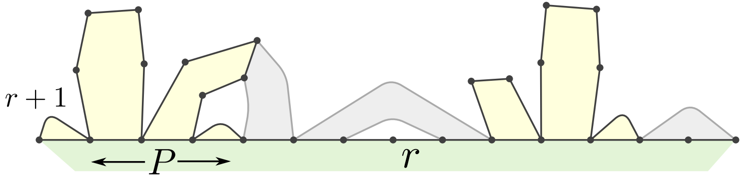

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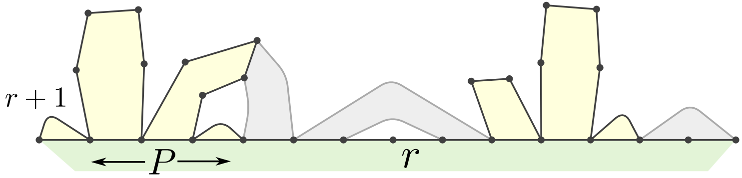

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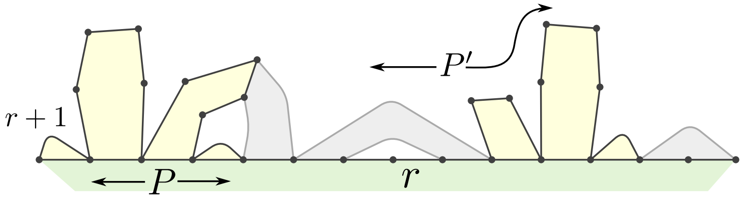
P'/P	$\approx e^{\mathcal{Z}}$ $(\mathbb{E}\mathcal{Z} > 0)$	≈ 1	≈ 1



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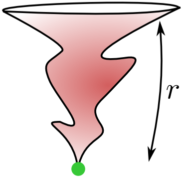
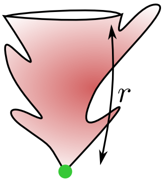
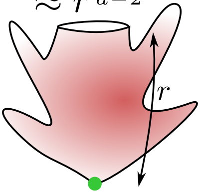
Perimeter at distance r	$\approx e^{cr}$ 	$\approx e^{\pi\sqrt{2}\sqrt{r}}$ 	$\approx r^{\frac{1}{a-2}}$ 

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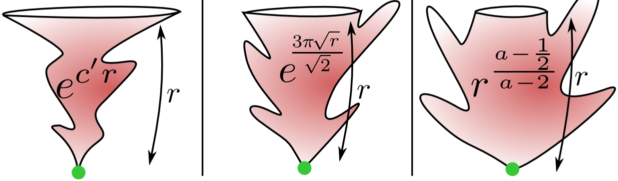
Volume of ball of radius r	$e^{c'r}$	$e^{\frac{3\pi\sqrt{r}}{\sqrt{2}}}$	$r^{\frac{a-\frac{1}{2}}{a-2}}$



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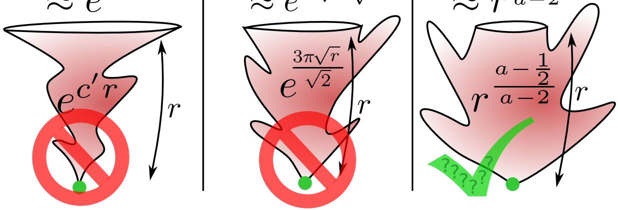
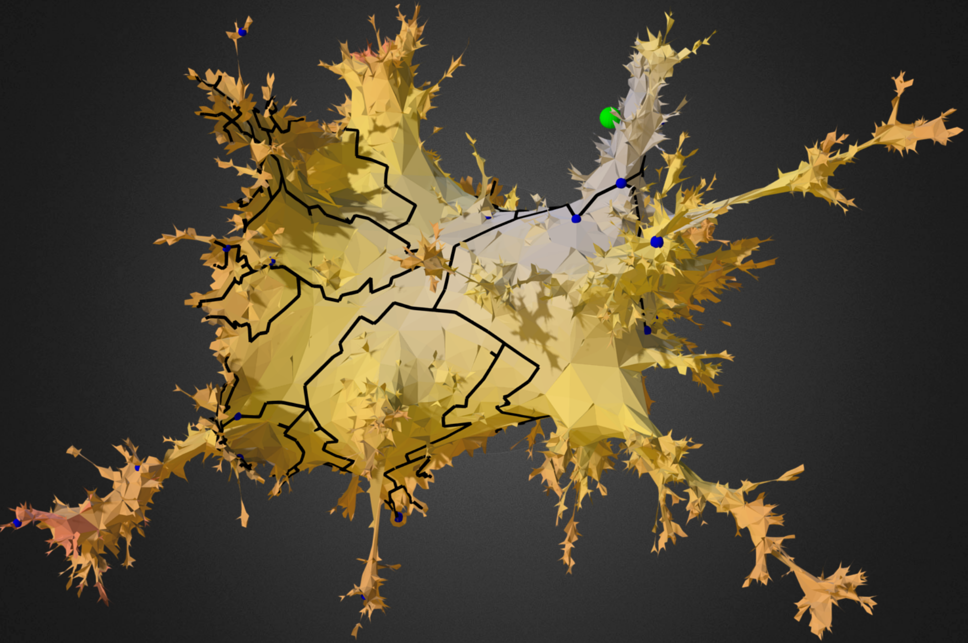
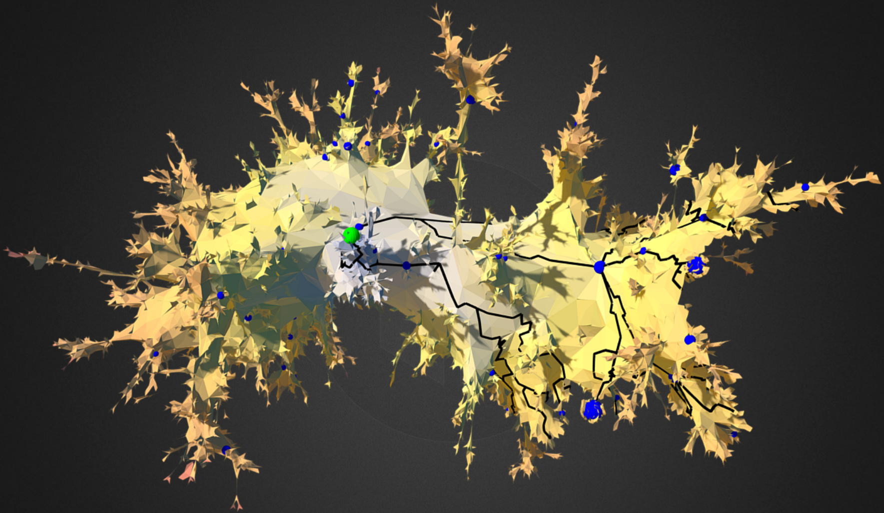
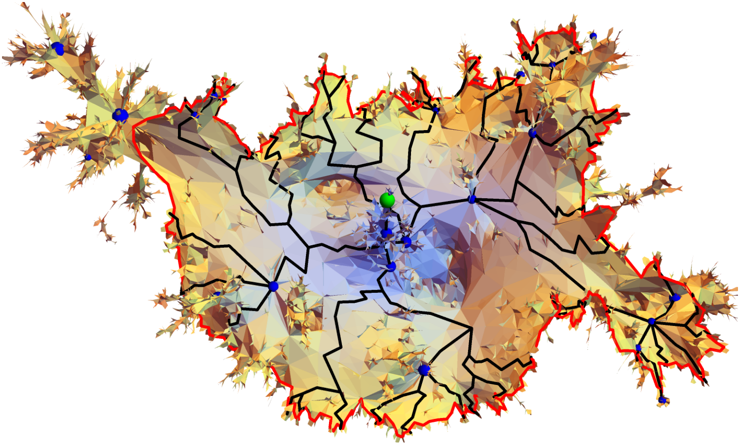
Scaling limit			







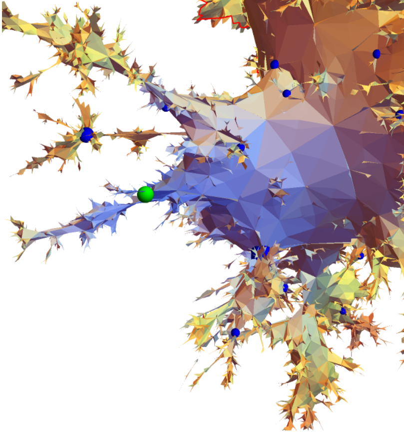
Geodesics



- Indeed geodesics like to merge in vertices of high degree! Hence not Brownian geometry!
- If scaling limit exists, $d_H = \frac{a-\frac{1}{2}}{a-2} > 4$.

Spectral properties

- The simple random walk on $\mathfrak{m}$ (with large degree faces) is always recurrent [Björnberg, Stefánsson]



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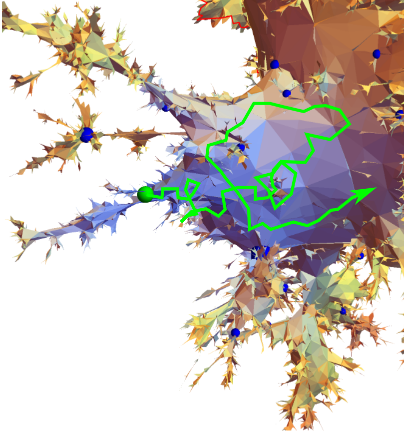

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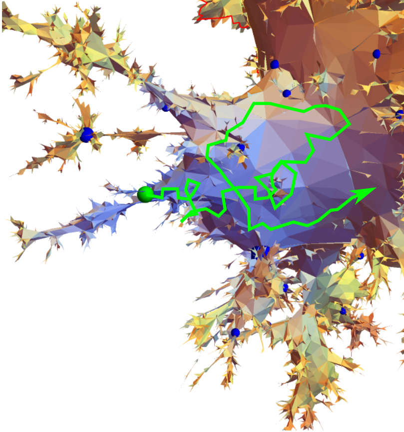

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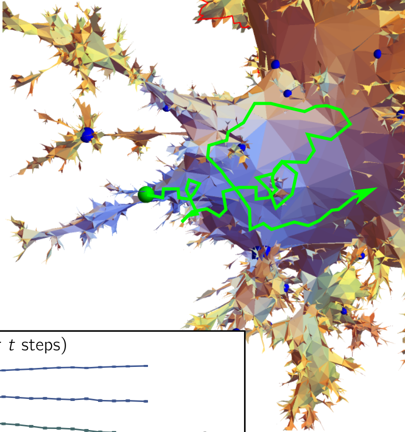

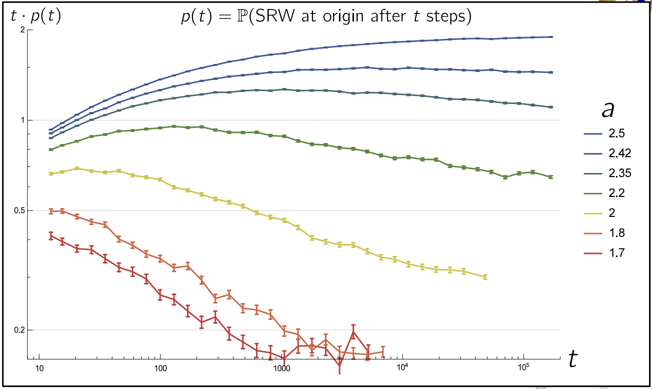

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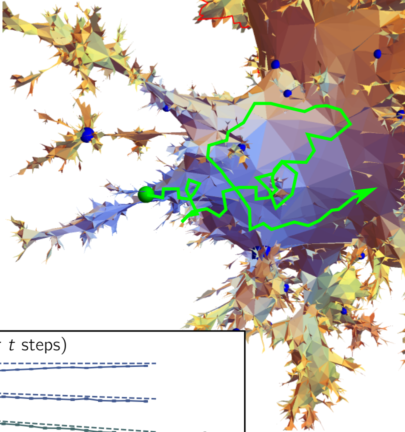

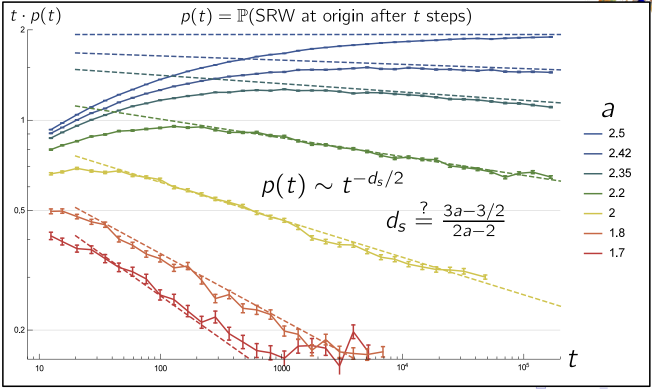

Questions

- Does "stable geometry" with $a \in (2, \frac{5}{2})$ form a new family of universality classes extending Brownian geometry ($a \to \frac{5}{2}$)?
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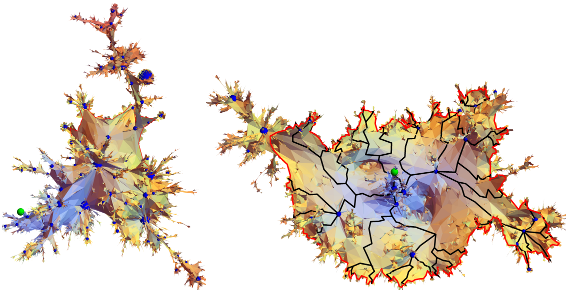
- Can the uniqueness conditions of Miller–Sheffield be weakened to single out the family of stable spheres?



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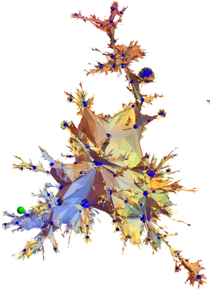

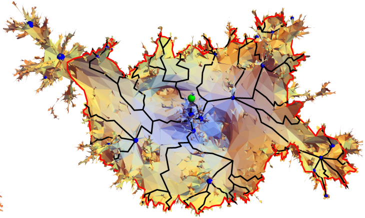

Thanks for your attention!