The effective kinetic term in CDT

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collaboration with Renate Loll

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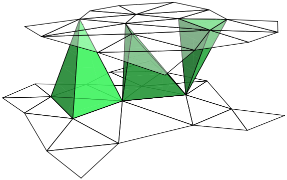

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Outline

- Brief intro to CDT in 2+1 dimensions
- CDT on a sphere
- Conformal mode problem
- Possible solution
- Test 1: Moduli measurement for CDT on torus
- Test 2: Extrinsic curvature at a boundary
- Conclusions

Introduction to CDT in 2+1 dimensions

- Causal Dynamical Triangulation is a regularization of the Euclidean path integral over geometries,

$$Z = \int [\mathcal{D}g] e^{-S[g]} \quad \rightarrow \quad Z_{CDT} = \sum_{\text{triangulations}\, T} e^{-S_{CDT}[T]}.$$

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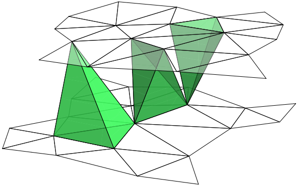

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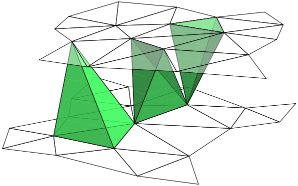


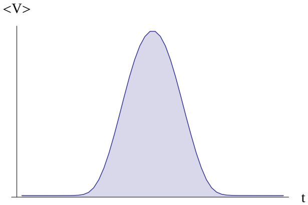
- Use Monte Carlo techniques to evaluate expectation values.

CDT with spatial topology of a sphere

- Well described by

$$S_{eff}[V] = \int dt \left(c_0 \frac{\dot{V}^2}{V} - c_1 V \right),$$

with $c_0, c_1 > 0$.



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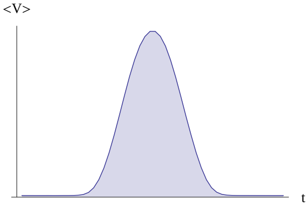

- If we evaluate Euclidean Einstein-Hilbert action $\int d^3x \sqrt{g}(-R + 2\Lambda)$ on spherical cosmology $ds^2 = dt^2 + V(t) d\Omega^2$,

$$S_{EH}[V] = -\kappa \int dt \left(\frac{\dot{V}^2}{V} - 2\Lambda V \right). \qquad (1)$$

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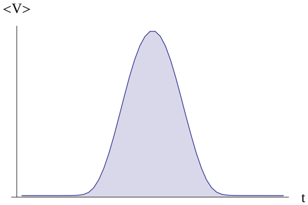

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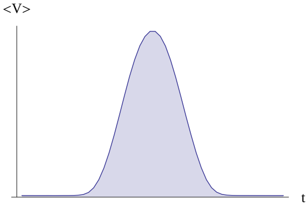

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- S_{eff} is bounded below (for fixed 3-volume), S_{EH} is not.

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$$S_{EH} = \kappa \int dt \int d^2x \sqrt{g} \left(\frac{1}{4} \dot{g}_{ab} \mathcal{G}^{abcd} \dot{g}_{cd} - R + 2\Lambda \right) \tag{2}$$

where $\mathcal{G}^{abcd}$ is the Wheeler-DeWitt metric,

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- Indefinite metric! Positive definite on traceless directions, negative definite on trace/conformal direction in superspace.
- CDT is a (well-defined) statistical system, therefore it better be described by a bounded action!
- Need some alternative to EH to compare too.

Solution: consider different kinetic term?

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$$\mathcal{G}_{\lambda}^{abcd} = \frac{1}{2}\left(g^{ac}g^{bd} + g^{ad}g^{bc}\right) - \lambda g^{ab}g^{cd}. \tag{4}$$

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- Two independent test of this ansatz.

CDT with spatial topology of the torus

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- Spatial volume $V(t)$ is a conformal degree of freedom. Need an observable measuring a traceless degree of freedom: measuring shape.

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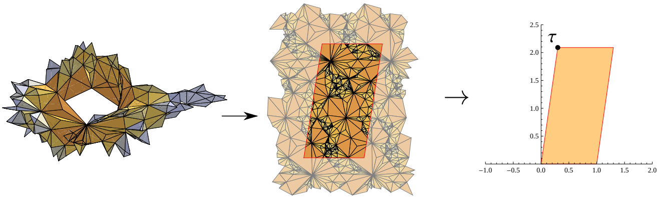
- Torus! There is a 2 parameter family of conformal equivalence classes of metrics on the torus, parametrized by the moduli parameter $\tau = \tau_1 + i\tau_2$.
- Given a 2D triangulation of the torus, we can find τ by constructing a periodic harmonic embedding in the plane.



- The modified kinetic term restricted to V and τ reads,

$$\kappa \int dt \left((\frac{1}{2} - \lambda) \frac{\dot{V}^2}{V} + \frac{1}{2A[g]} \frac{\dot{\tau}_1^2 + \dot{\tau}_2^2}{\tau_2^2} \right), \tag{5}$$

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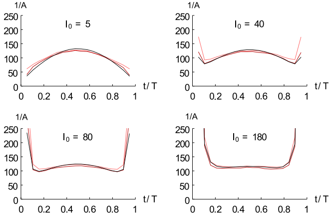

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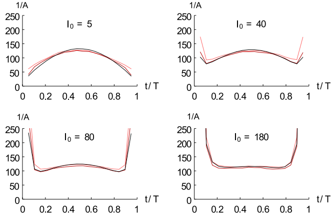

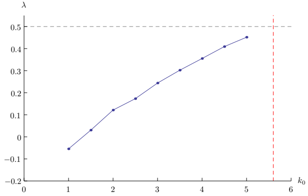

Extrinsic curvature at a boundary

- To test our ansatz more locally we consider the extrinsic curvature at a fixed spatial boundary. According to our ansatz

$$\langle K_{ab}(x) K_{cd}(y) \rangle - \langle K_{ab}(x) \rangle \langle K_{cd}(y) \rangle \propto \delta(x-y) \mathcal{G}^{(\lambda)}_{abcd}. \qquad (7)$$

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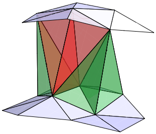

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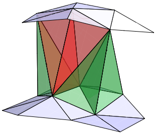

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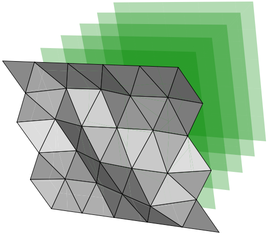

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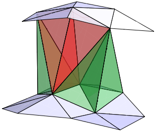

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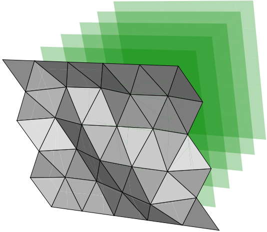

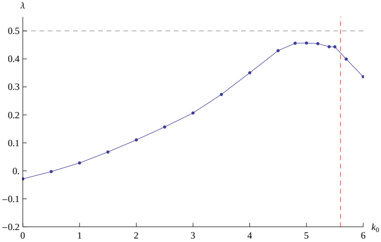

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These and other results to appear on arXiv soon.

Thanks!