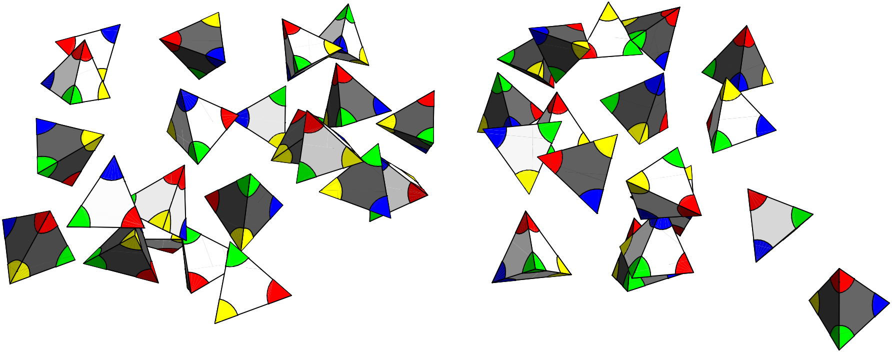
Adding colors to dynamical triangulations in 3d

Timothy Budd
budd@nbi.dk, office: Fb-11



September 20, 2012
Niels Bohr Institute

(Euclidean) Dynamical Triangulations in 3d

- Central object in Euclidean path integral approach to quantum gravity:

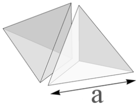
$$Z_{QG} = \int_{S^3} \frac{\mathcal{D}g}{\mathrm{Diff}} e^{-S_{EH}[g]}$$

(Euclidean) Dynamical Triangulations in 3d

- Central object in Euclidean path integral approach to quantum gravity:

$$Z_{QG} = \int_{S^3} \frac{\mathcal{D}g}{\mathrm{Diff}} e^{-S_{EH}[g]} \quad \xrightarrow[\text{in DT}]{} \quad Z_{DT} = \sum_T \frac{1}{C_T} e^{-S_{DT}[T]}$$

- DT is a lattice regularization replacing the integral by a sum over triangulations built from N_3 equilateral tetrahedra.

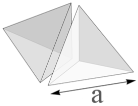


(Euclidean) Dynamical Triangulations in 3d

- Central object in Euclidean path integral approach to quantum gravity:

$$Z_{QG} = \int_{S^3} \frac{\mathcal{D}g}{\mathrm{Diff}} e^{-S_{EH}[g]} \quad \xrightarrow[\text{in DT}]{} \quad Z_{DT} = \sum_T \frac{1}{C_T} e^{-S_{DT}[T]}$$

- DT is a lattice regularization replacing the integral by a sum over triangulations built from N_3 equilateral tetrahedra.



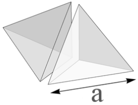
$$S_{DT}[T] = k_3 N_3 - k_0 N_0, \quad k_3 \sim \Lambda, k_0 \sim 1/G.$$

(Euclidean) Dynamical Triangulations in 3d

- Central object in Euclidean path integral approach to quantum gravity:

$$Z_{QG} = \int_{S^3} \frac{\mathcal{D}g}{\mathrm{Diff}} e^{-S_{EH}[g]} \xrightarrow[\text{in DT}]{} Z_{DT} = \sum_T \frac{1}{C_T} e^{-S_{DT}[T]}$$

- DT is a lattice regularization replacing the integral by a sum over triangulations built from N_3 equilateral tetrahedra.



$$S_{DT}[T] = k_3 N_3 - k_0 N_0, \quad k_3 \sim \Lambda, k_0 \sim 1/G.$$

- Number of triangulations of fixed topology grows exponentially with N_3. Therefore a critical coupling $k_3^*(k_0)$ exists s.t.

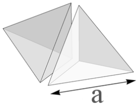
$$\langle N_3 \rangle < \infty \text{ for } k_3 < k_3^* \quad \text{and } Z \text{ diverges for } k_3 > k_3^*$$

(Euclidean) Dynamical Triangulations in 3d

- Central object in Euclidean path integral approach to quantum gravity:

$$Z_{QG} = \int_{S^3} \frac{\mathcal{D}g}{\mathrm{Diff}} e^{-S_{EH}[g]} \xrightarrow[\text{in DT}]{} Z_{DT} = \sum_T \frac{1}{C_T} e^{-S_{DT}[T]}$$

- DT is a lattice regularization replacing the integral by a sum over triangulations built from N_3 equilateral tetrahedra.



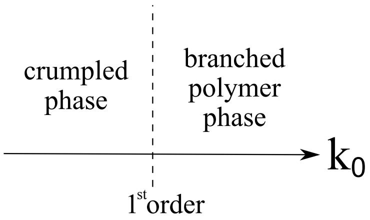
$$S_{DT}[T] = k_3 N_3 - k_0 N_0, \quad k_3 \sim \Lambda, k_0 \sim 1/G.$$

- Number of triangulations of fixed topology grows exponentially with N_3. Therefore a critical coupling $k_3^*(k_0)$ exists s.t.

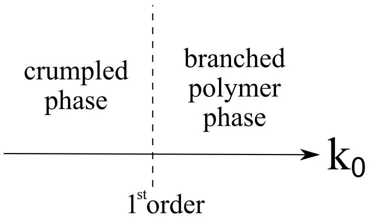
$$\langle N_3 \rangle < \infty \text{ for } k_3 < k_3^* \quad \text{and } Z \text{ diverges for } k_3 > k_3^*$$

- To find a continuum limit we need $N_3 \to \infty$, which can be achieved by tuning $k_3 \to k_3^*$, but also lattice spacing $a \to 0$. For this we need a phase transition.

- Phase diagram of 3d DT in infinite volume limit.

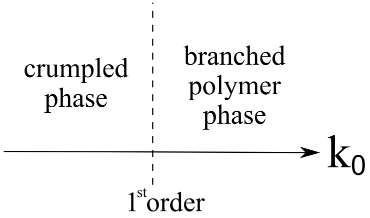


- Phase diagram of 3d DT in infinite volume limit.



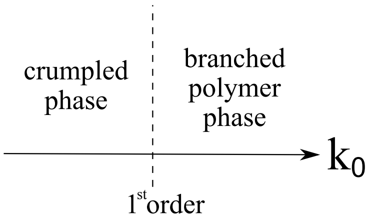
- Issues with DT:
  - Both phases have little to do with 3d geometry, e.g. Hausdorff dimensions ∞ resp. 2.

- Phase diagram of 3d DT in infinite volume limit.



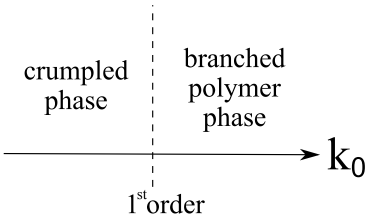
- Issues with DT:
  - Both phases have little to do with 3d geometry, e.g. Hausdorff dimensions ∞ resp. 2.
  - Phase transition too strong for continuum limit.

- Phase diagram of 3d DT in infinite volume limit.



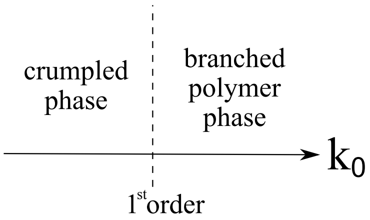
- Issues with DT:
  - Both phases have little to do with 3d geometry, e.g. Hausdorff dimensions ∞ resp. 2.
  - Phase transition too strong for continuum limit.
  - No analytic tools available. We have to rely on Monte Carlo simulations.

- Phase diagram of 3d DT in infinite volume limit.



- Issues with DT:
  - Both phases have little to do with 3d geometry, e.g. Hausdorff dimensions ∞ resp. 2.
  - Phase transition too strong for continuum limit.
  - No analytic tools available. We have to rely on Monte Carlo simulations.
- Various attempts to cure some of these issues (90's): Modified actions, matter fields, ... (CDT)

- Phase diagram of 3d DT in infinite volume limit.



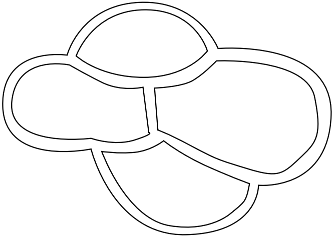
- Issues with DT:
  - Both phases have little to do with 3d geometry, e.g. Hausdorff dimensions ∞ resp. 2.
  - Phase transition too strong for continuum limit.
  - No analytic tools available. We have to rely on Monte Carlo simulations.
- Various attempts to cure some of these issues (90's): Modified actions, matter fields, ... (CDT)
- This talk: new model which has not been studied before from DT perspective (to my knowledge).

Outline

- Motivation for colored DT: matrix models $\rightarrow$ tensor models $\rightarrow$ colored tensor models $\rightarrow$ colored DT
- Monte Carlo simulations
- Analytic approach to branched polymer phase
- Conclusions & Outlook

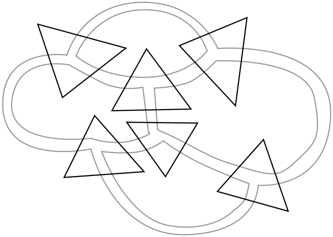
Matrix models

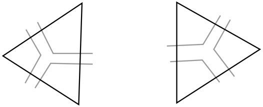
- Typical matrix model: $Z = \int dX_{ij} e^{-N\left(\mathrm{Tr}[X^2] + \lambda \mathrm{Tr}[X^3]\right)}$, $1 \leq i, j \leq N$.



Matrix models

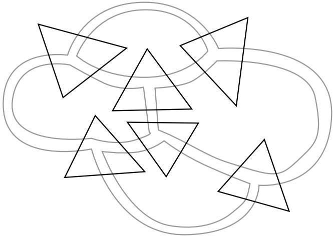
- Typical matrix model: $Z = \int dX_{ij} e^{-N\left(\mathrm{Tr}[X^2]+\lambda \mathrm{Tr}[X^3]\right)}$, $1 \leq i, j \leq N$.

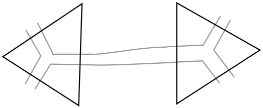




Matrix models

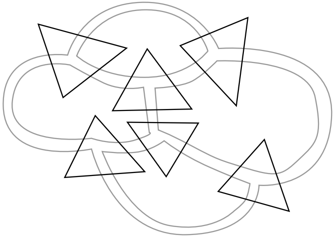
- Typical matrix model: $Z = \int dX_{ij} e^{-N\left(\mathrm{Tr}[X^2]+\lambda \mathrm{Tr}[X^3]\right)}$, $1 \leq i,j \leq N$.

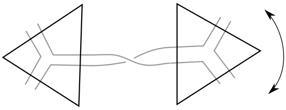




Matrix models

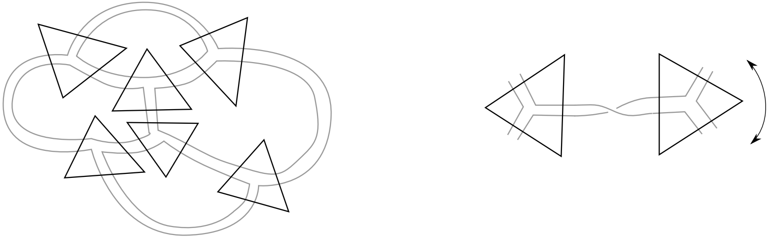
- Typical matrix model: $Z = \int dX_{ij} e^{-N\left(\mathrm{Tr}[X^2]+\lambda \mathrm{Tr}[X^3]\right)}$, $1 \le i, j \le N$.





Matrix models

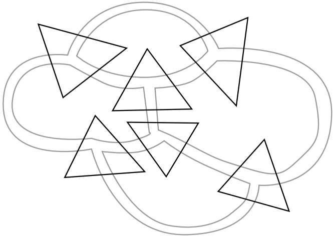
- Typical matrix model: $Z = \int dX_{ij} e^{-N\left(\mathrm{Tr}[X^2]+\lambda \mathrm{Tr}[X^3]\right)}$, $1 \le i, j \le N$.

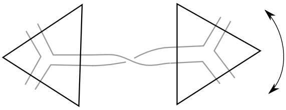


- Amplitude $N^{-E}(\lambda N)^V N^F = N^{V-E+F}\lambda^V = N^{\chi}\lambda^V = N^{2-2g}\lambda^V$.

Matrix models

- Typical matrix model: $Z = \int dX_{ij} e^{-N\left(\mathrm{Tr}[X^2]+\lambda \mathrm{Tr}[X^3]\right)}$, $1 \leq i,j \leq N$.

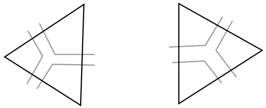




- Amplitude $N^{-E}(\lambda N)^V N^F = N^{V-E+F}\lambda^V = N^{\chi}\lambda^V = N^{2-2g}\lambda^V$.
- For $N \to \infty$ planar triangulations dominate ($g = 0$).

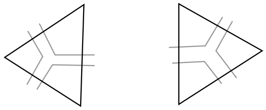
Tensor models

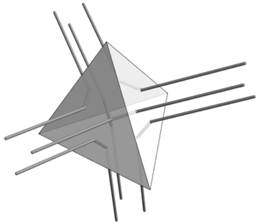
- Straightforward generalization:
  $Z = \int dX_{ijk} e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk} X^{i}_{lm} X^{jl}_{n} X^{kmn}\right)}$.



Tensor models

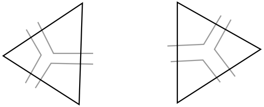
- Straightforward generalization:
  $Z = \int dX_{ijk}\, e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk} X^{i}_{lm} X^{jl}_{n} X^{kmn}\right)}.$

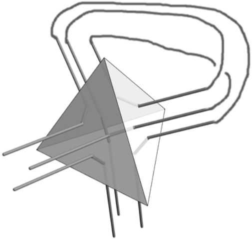




Tensor models

- Straightforward generalization:
  $Z = \int dX_{ijk}\, e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk} X^{i}_{lm} X^{jl}_{n} X^{kmn}\right)}$.

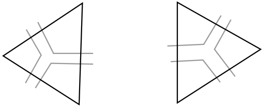


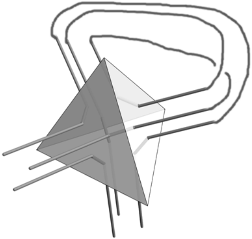


- Contrary to 2D most gluings do not give (simplicial) manifolds.
- Large N limit?

Tensor models

- Straightforward generalization:
  $Z = \int dX_{ijk} e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk}X^{i}_{lm}X^{jl}_{n}X^{kmn}\right)}$.





- Contrary to 2D most gluings do not give (simplicial) manifolds.
- Large N limit?
- Moreover, a lot of information to keep track of.

Colored tensor models [Gurau, Bonzom]

- Instead of $Z = \int dX_{ijk} e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk}X^{i}_{lm}X^{jl}_{n}X^{kmn}\right)}$, we can take 4 complex tensors of different colors (index contraction is implied):

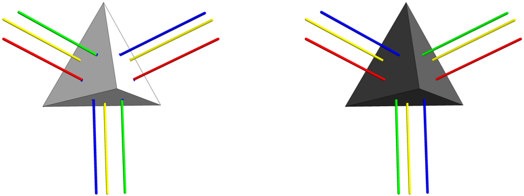
$$Z_{\mathrm{CTM}} = \int dX\, dX\, dX\, dX\, e^{-N^{3/2}\left(X\bar{X} + X\bar{X} + X\bar{X} + X\bar{X} + \lambda XXXX + \lambda \bar{X}\bar{X}\bar{X}\bar{X}\right)}$$

Colored tensor models [Gurau, Bonzom]

- Instead of $Z = \int dX_{ijk}\, e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk} X^{i}_{lm} X^{jl}_{n} X^{kmn}\right)}$, we can take 4 complex tensors of different colors (index contraction is implied):

$$Z_{\mathrm{CTM}} = \int dX\, dX\, dX\, dX\, e^{-N^{3/2}\left(X\bar{X} + X\bar{X} + X\bar{X} + X\bar{X} + \lambda XXXX + \lambda \bar{X}\bar{X}\bar{X}\bar{X}\right)}$$

- Get two vertices with opposite orientation (white and black)

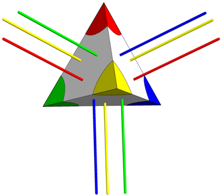


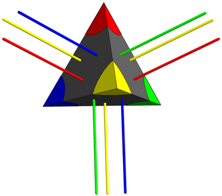
Colored tensor models [Gurau, Bonzom]

- Instead of $Z = \int dX_{ijk}\, e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk}X^{i}_{lm}X^{jl}_{n}X^{kmn}\right)}$, we can take 4 complex tensors of different colors (index contraction is implied):

$$Z_{\mathrm{CTM}} = \int dX\, dX\, dX\, dX\, e^{-N^{3/2}\left(X\bar{X} + X\bar{X} + X\bar{X} + X\bar{X} + \lambda XXXX + \lambda \bar{X}\bar{X}\bar{X}\bar{X}\right)}$$

- Get two vertices with opposite orientation (white and black)





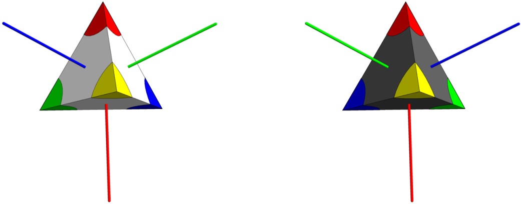
- We are dealing with colored tetrahedra and can forget about the strands!

Colored tensor models [Gurau, Bonzom]

- Instead of $Z = \int dX_{ijk} e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk} X^{i}_{lm} X^{jl}_{n} X^{kmn}\right)}$, we can take 4 complex tensors of different colors (index contraction is implied):

$$Z_{\mathrm{CTM}} = \int dX\, dX\, dX\, dX\, e^{-N^{3/2}\left(X\bar{X} + X\bar{X} + X\bar{X} + X\bar{X} + \lambda XXXX + \lambda \bar{X}\bar{X}\bar{X}\bar{X}\right)}$$

- Get two vertices with opposite orientation (white and black)



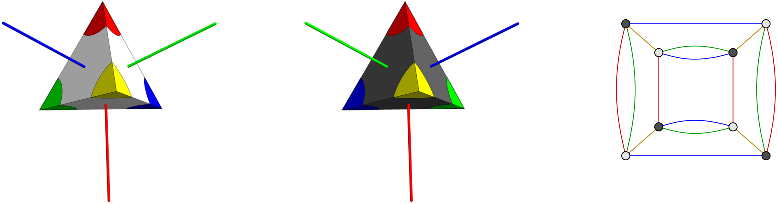
- We are dealing with colored tetrahedra and can forget about the strands!

Colored tensor models [Gurau, Bonzom]

- Instead of $Z = \int dX_{ijk} e^{-N^{3/2}\left(X_{ijk}X^{ijk} + \lambda X_{ijk} X^{i}_{lm} X^{jl}_{n} X^{kmn}\right)}$, we can take 4 complex tensors of different colors (index contraction is implied):

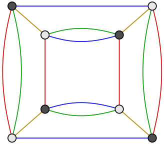
$$Z_{\text{CTM}} = \int dX\, dX\, dX\, dX\, e^{-N^{3/2}\left(X\bar{X} + X\bar{X} + X\bar{X} + X\bar{X} + \lambda XXXX + \lambda \bar{X}\bar{X}\bar{X}\bar{X}\right)}$$

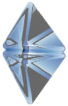
- Get two vertices with opposite orientation (white and black)



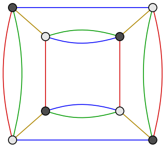
- We are dealing with colored tetrahedra and can forget about the strands! The gluing of tetrahedra is uniquely encoded in bipartite colored graph!

- Every bipartite colored graph corresponds to a pseudo simplicial manifold, a manifold with mild topological singularities.

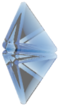


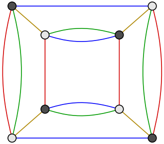


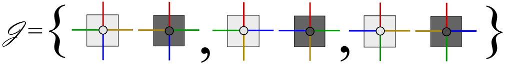
- Every bipartite colored graph corresponds to a pseudo simplicial manifold, a manifold with mild topological singularities.
- Amplitude is $\lambda^V N^{3-\omega}$, where the 'degree' ω is a generalization of genus g to higher dimensions. Given in terms of the genus of its 'jackets': $\omega = \sum_{\mathcal{J}} g_J \geq 0$.

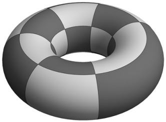


- Every bipartite colored graph corresponds to a pseudo simplicial manifold, a manifold with mild topological singularities.
- Amplitude is $\lambda^V N^{3-\omega}$, where the 'degree' ω is a generalization of genus g to higher dimensions. Given in terms of the genus of its 'jackets': $\omega = \sum_{\mathcal{J}} g_J \geq 0$.

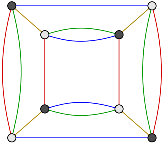




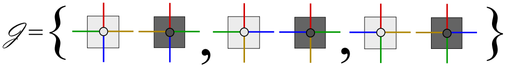


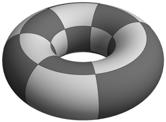


- Every bipartite colored graph corresponds to a pseudo simplicial manifold, a manifold with mild topological singularities.

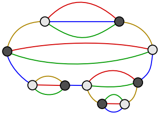


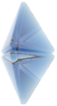
- Amplitude is $\lambda^V N^{3-\omega}$, where the 'degree' ω is a generalization of genus g to higher dimensions. Given in terms of the genus of its 'jackets': $\omega = \sum_{\mathcal{J}} g_J \geq 0$.



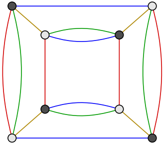


- ω is not a topological invariant, but $\omega = 0$ corresponds to special 'melonic' triangulations of S^3.

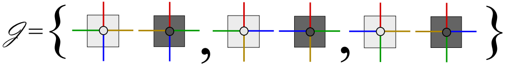


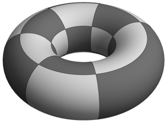


- Every bipartite colored graph corresponds to a pseudo simplicial manifold, a manifold with mild topological singularities.

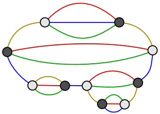


- Amplitude is $\lambda^V N^{3-\omega}$, where the 'degree' ω is a generalization of genus g to higher dimensions. Given in terms of the genus of its 'jackets': $\omega = \sum_{\mathcal{J}} g_J \geq 0$.





- ω is not a topological invariant, but $\omega = 0$ corresponds to special 'melonic' triangulations of S^3.
- Limit $N \to \infty$ well-defined in terms of melonic graphs.



- Double scaling limit to include $\omega > 0$? Control over topologies? Independent methods to evaluate tensor model? Still a long way to go.

Colored dynamical triangulations

- Partition function of colored tensor models

$$Z_{CTM}(\lambda, N) = \sum_{\text{colored graphs } \mathcal{G}} \frac{1}{C_{\mathcal{G}}} \lambda^V N^{3-\omega}$$

Colored dynamical triangulations

- Partition function of colored tensor models

$$Z_{CTM}(\lambda, N) = \sum_{\text{colored graphs } \mathcal{G}} \frac{1}{C_{\mathcal{G}}} \lambda^V N^{3-\omega}$$

- Restrict sum to graphs $\mathcal{G}$ with topology S^3 gives colored dynamical triangulations

$$Z^c_{\mathrm{DT}}(k_0, k_3) = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} \lambda^V N^{3-\omega} = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} e^{-k_3 N_3 + k_0 N_0}$$

$$N_3 = V, \quad 3 - \omega = N_0 - \frac{N_3}{2}, \quad \Rightarrow \quad e^{-k_3} = \frac{\lambda}{\sqrt{N}}, \quad e^{k_0} = N.$$

Colored dynamical triangulations

- Partition function of colored tensor models

$$Z_{CTM}(\lambda, N) = \sum_{\text{colored graphs } \mathcal{G}} \frac{1}{C_{\mathcal{G}}} \lambda^V N^{3-\omega}$$

- Restrict sum to graphs $\mathcal{G}$ with topology S^3 gives colored dynamical triangulations

$$Z^c_{\mathrm{DT}}(k_0, k_3) = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} \lambda^V N^{3-\omega} = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} e^{-k_3 N_3 + k_0 N_0}$$

$$N_3 = V, \quad 3 - \omega = N_0 - \frac{N_3}{2}, \quad \Rightarrow \quad e^{-k_3} = \frac{\lambda}{\sqrt{N}}, \quad e^{k_0} = N.$$

- Melons occur at $N \to \infty$, i.e. $k_0 \to \infty$.
  Typical structure of branched polymers.

Colored dynamical triangulations

- Partition function of colored tensor models

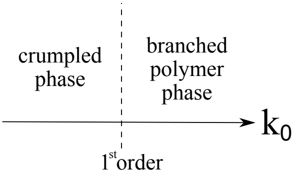
$$Z_{CTM}(\lambda, N) = \sum_{\text{colored graphs } \mathcal{G}} \frac{1}{C_{\mathcal{G}}} \lambda^V N^{3-\omega}$$

- Restrict sum to graphs $\mathcal{G}$ with topology S^3 gives colored dynamical triangulations

$$Z^c_{\mathrm{DT}}(k_0, k_3) = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} \lambda^V N^{3-\omega} = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} e^{-k_3 N_3 + k_0 N_0}$$

$$N_3 = V, \quad 3 - \omega = N_0 - \frac{N_3}{2}, \quad \Rightarrow \quad e^{-k_3} = \frac{\lambda}{\sqrt{N}}, \quad e^{k_0} = N.$$

- Melons occur at $N \to \infty$, i.e. $k_0 \to \infty$. Typical structure of branched polymers.
- Goal: examine $Z^c_{\mathrm{DT}}(k_0, k_3)$ at finite k_0

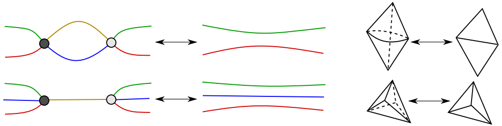


Monte Carlo simulations

- $Z^{c}_{\mathrm{DT}}(k_0, k_3) = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} e^{-k_3 N_3 + k_0 N_0}$ defines a statistical ensemble with Boltzmann weights $e^{-k_3 N_3 + k_0 N_0}$.

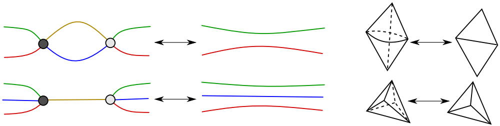
Monte Carlo simulations

- $Z^c_{\mathrm{DT}}(k_0, k_3) = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} e^{-k_3 N_3 + k_0 N_0}$ defines a statistical ensemble with Boltzmann weights $e^{-k_3 N_3 + k_0 N_0}$.
- Generate set $\{\mathcal{G}_i\}$ of random triangulation by applying large number of random 'dipole moves' on initial triangulation. Observables $\langle \mathcal{O} \rangle_{Z^c_{\mathrm{DT}}} \approx \frac{1}{n} \sum_{i=1}^{n} \mathcal{O}(\mathcal{G}_i)$.

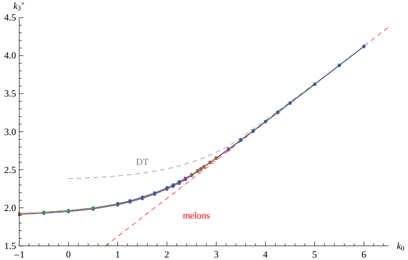


Monte Carlo simulations

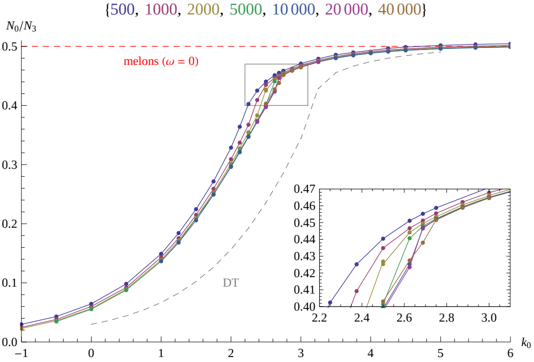
- $Z^c_{\mathrm{DT}}(k_0,k_3) = \sum_{\mathcal{G}_{S^3}} \frac{1}{C_{\mathcal{G}}} e^{-k_3 N_3 + k_0 N_0}$ defines a statistical ensemble with Boltzmann weights $e^{-k_3 N_3 + k_0 N_0}$.
- Generate set $\{\mathcal{G}_i\}$ of random triangulation by applying large number of random 'dipole moves' on initial triangulation. Observables $\langle \mathcal{O} \rangle_{Z^c_{\mathrm{DT}}} \approx \frac{1}{n}\sum_{i=1}^{n} \mathcal{O}(\mathcal{G}_i)$.



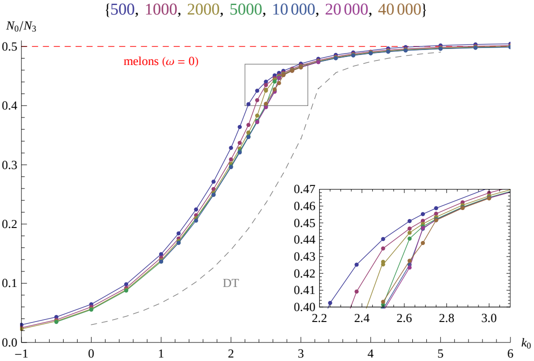
- To get large N_3 we need to tune k_3 to critical coupling $k_3^*(k_0)$.



- Measure the observable $\langle N_0 \rangle / \langle N_3 \rangle$

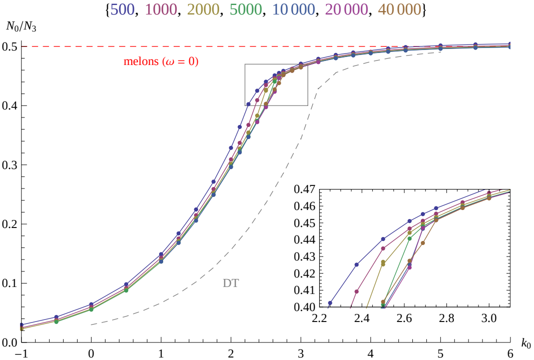


- Measure the observable $\langle N_0 \rangle / \langle N_3 \rangle$

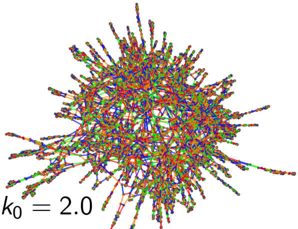


- Phase transition seems to be present at $k_0 \approx 2.7$.

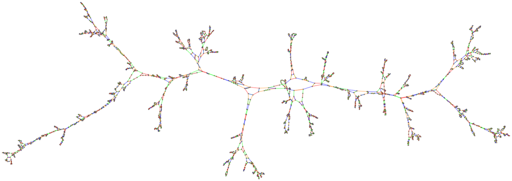
- Measure the observable $\langle N_0 \rangle / \langle N_3 \rangle$



- Phase transition seems to be present at $k_0 \approx 2.7$.



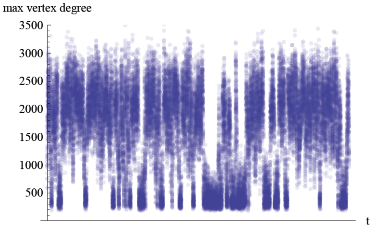
$k_0 = 2.0$

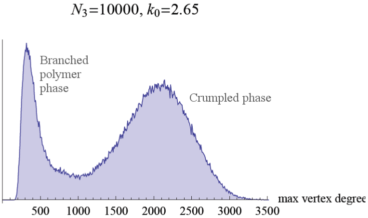


$k_0 = 3.5$

Order of the phase transition

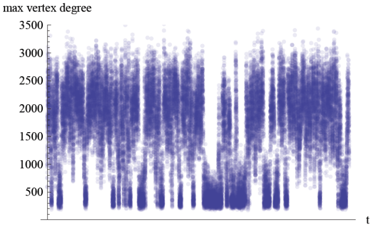
- Try to find double peak structure in histogram of some order parameter by tuning k_0 to phase transition. A convenient choice of order parameter is the maximal vertex degree (the degree of a vertex is the number of tetrahedra sharing it).

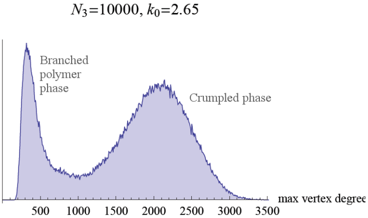




Order of the phase transition

- Try to find double peak structure in histogram of some order parameter by tuning k_0 to phase transition. A convenient choice of order parameter is the maximal vertex degree (the degree of a vertex is the number of tetrahedra sharing it).

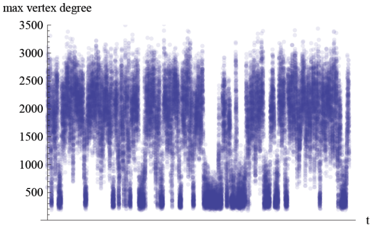


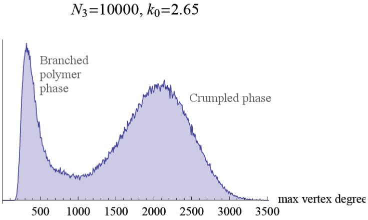


- If the peaks become more distinct as N_3 increases, the phase transition is 1^{st} order.

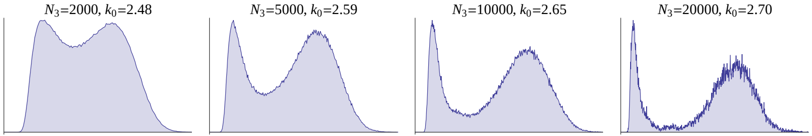
Order of the phase transition

- Try to find double peak structure in histogram of some order parameter by tuning k_0 to phase transition. A convenient choice of order parameter is the maximal vertex degree (the degree of a vertex is the number of tetrahedra sharing it).





- If the peaks become more distinct as N_3 increases, the phase transition is 1^{st} order. Check!

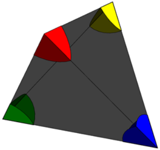


- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.

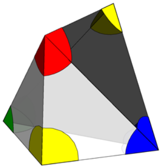
- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, . . .

- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, . . .
- This suggests colored DT and uncolored DT in same universality class.

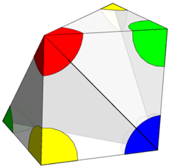
- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, ...
- This suggests colored DT and uncolored DT in same universality class.
- Not unexpected, but not obvious either. Colors put a severe restriction on triangulations.
- Not even clear a priori whether 3d extended colored triangulations exist. However, here is an example:



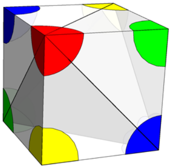
- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, ...
- This suggests colored DT and uncolored DT in same universality class.
- Not unexpected, but not obvious either. Colors put a severe restriction on triangulations.
- Not even clear a priori whether 3d extended colored triangulations exist. However, here is an example:



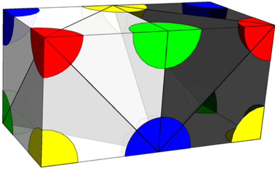
- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, ...
- This suggests colored DT and uncolored DT in same universality class.
- Not unexpected, but not obvious either. Colors put a severe restriction on triangulations.
- Not even clear a priori whether 3d extended colored triangulations exist. However, here is an example:



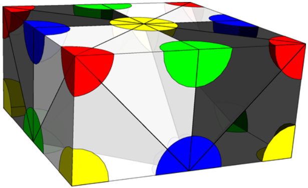
- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, ...
- This suggests colored DT and uncolored DT in same universality class.
- Not unexpected, but not obvious either. Colors put a severe restriction on triangulations.
- Not even clear a priori whether 3d extended colored triangulations exist. However, here is an example:



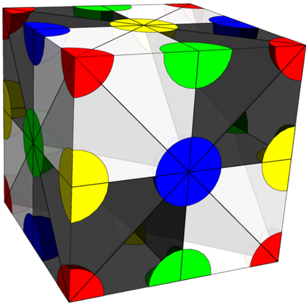
- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, ...
- This suggests colored DT and uncolored DT in same universality class.
- Not unexpected, but not obvious either. Colors put a severe restriction on triangulations.
- Not even clear a priori whether 3d extended colored triangulations exist. However, here is an example:



- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, ...
- This suggests colored DT and uncolored DT in same universality class.
- Not unexpected, but not obvious either. Colors put a severe restriction on triangulations.
- Not even clear a priori whether 3d extended colored triangulations exist. However, here is an example:



- Phase diagram of colored DT very similar to uncolored DT, both have 1^{st} order phase transition.
- Agreement on various characteristics of both phases: Hausdorff dimension, spectral dimension, susceptibility exponent, ...
- This suggests colored DT and uncolored DT in same universality class.
- Not unexpected, but not obvious either. Colors put a severe restriction on triangulations.
- Not even clear a priori whether 3d extended colored triangulations exist. However, here is an example:



Towards analytic evaluation at finite k_0

- At $e^{-k_0} = 0$ ($N \to \infty$) the partition function Z^c_{DT} is dominated by melonic triangulations, which can be easily summed. Can we go beyond $e^{-k_0} = 0$?

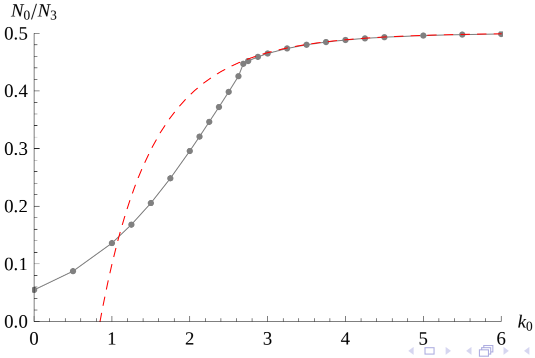
Towards analytic evaluation at finite k_0

- At $e^{-k_0} = 0$ ($N \to \infty$) the partition function Z^c_{DT} is dominated by melonic triangulations, which can be easily summed. Can we go beyond $e^{-k_0} = 0$?
- Yes, we can in principle evaluate Z^c_{DT} order by order in e^{-k_0}.

Towards analytic evaluation at finite k_0

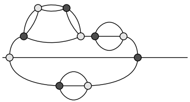
- At $e^{-k_0} = 0$ ($N \to \infty$) the partition function Z^c_{DT} is dominated by melonic triangulations, which can be easily summed. Can we go beyond $e^{-k_0} = 0$?
- Yes, we can in principle evaluate Z^c_{DT} order by order in e^{-k_0}.
- Sneak preview:

$$\frac{\langle \omega \rangle}{\langle N_3 \rangle} = \frac{5}{4} e^{-k_0} + \frac{81}{32} e^{-2k_0} + \cdots, \frac{\langle N_0 \rangle}{\langle N_3 \rangle} = \frac{1}{2} - \frac{1}{2} \frac{\langle \omega \rangle}{\langle N_3 \rangle} \quad (N_3 \to \infty)$$



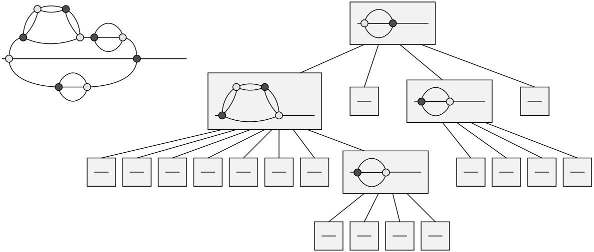
Relation to the 2PI partition function

- Let's consider two-point function $G = 1 - \frac{1}{2}\frac{\partial Z^{c}_{\mathrm{DT}}(k_3, k_0)}{\partial k_3}$.



Relation to the 2PI partition function

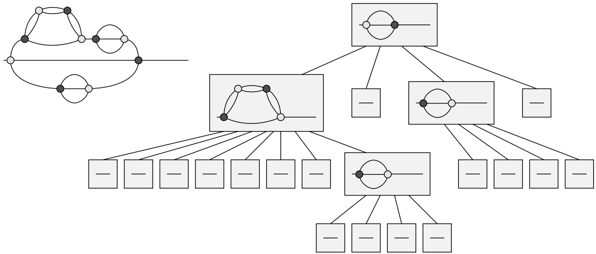
- Let's consider two-point function $G = 1 - \frac{1}{2}\frac{\partial Z^c_{\mathrm{DT}}(k_3,k_0)}{\partial k_3}$.



- Unique decomposition into 2 Particle Irreducible (2PI) two-point graphs.

Relation to the 2PI partition function

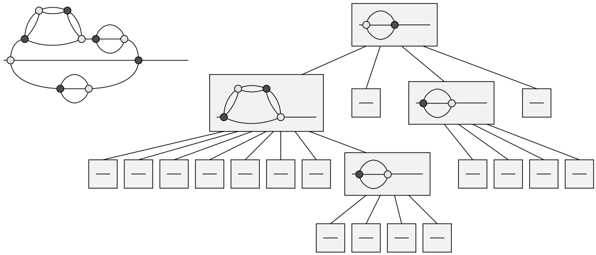
- Let's consider two-point function $G = 1 - \frac{1}{2}\frac{\partial Z^{c}_{\mathrm{DT}}(k_3,k_0)}{\partial k_3}$.



- Unique decomposition into 2 Particle Irreducible (2PI) two-point graphs.
- The ensemble corresponding to $G(k_3, k_0)$ is that of random trees with offspring distribution governed by the 2PI two-point function $G_{2PI}(\bar{k}_3, k_0)$.

Relation to the 2PI partition function

- Let's consider two-point function $G = 1 - \frac{1}{2}\frac{\partial Z^c_{\mathrm{DT}}(k_3,k_0)}{\partial k_3}$.



- Unique decomposition into 2 Particle Irreducible (2PI) two-point graphs.
- The ensemble corresponding to $G(k_3, k_0)$ is that of random trees with offspring distribution governed by the 2PI two-point function $G_{2PI}(\bar{k}_3, k_0)$.
- We are interested in $N_3 \to \infty$, hence in so-called critical trees which have $\langle \text{offspring number} \rangle = 1$, i.e. $\langle N_3 \rangle_{G_{2PI}} = \frac{1}{2}$.

- It follows that $G(k_3 = k_3^*, k_0)$ is directly related to $G_{2PI}(\bar{k}_3, k_0)$ with $\bar{k}_3$ chosen such that $\langle N_3 \rangle_{G_{2PI}} = 1/2$.

- It follows that $G(k_3 = k_3^*, k_0)$ is directly related to $G_{2PI}(\bar{k}_3, k_0)$ with $\bar{k}_3$ chosen such that $\langle N_3 \rangle_{G_{2PI}} = 1/2$.
- In particular

$$\left. \frac{\langle \omega \rangle}{\langle N_3 \rangle} \right|_{k_3 = k_3^*} = \left. \frac{\langle \omega \rangle_{2PI}}{\langle N_3 \rangle_{2PI}} \right|_{\langle N_3 \rangle_{2PI} = 1/2} = 2 \left. \langle \omega \rangle_{2PI} \right|_{\langle N_3 \rangle_{2PI} = 1/2}$$

- It follows that $G(k_3 = k_3^*, k_0)$ is directly related to $G_{2PI}(\bar{k}_3, k_0)$ with $\bar{k}_3$ chosen such that $\langle N_3 \rangle_{G_{2PI}} = 1/2$.
- In particular

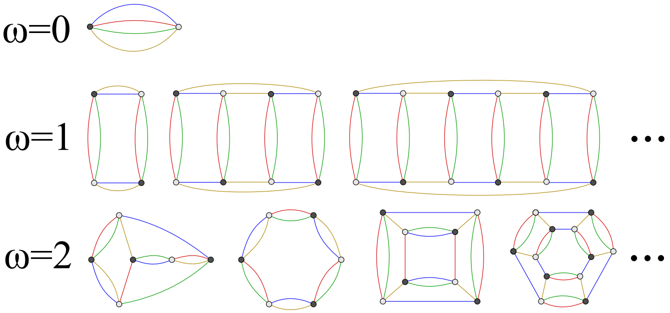
$$\frac{\langle \omega \rangle}{\langle N_3 \rangle}\bigg|_{k_3 = k_3^*} = \frac{\langle \omega \rangle_{2PI}}{\langle N_3 \rangle_{2PI}}\bigg|_{\langle N_3 \rangle_{2PI} = 1/2} = 2 \, \langle \omega \rangle_{2PI}\big|_{\langle N_3 \rangle_{2PI} = 1/2}$$

- To find $\langle \omega \rangle_{2PI}$ up to order $(e^{-k_0})^n$ we only need the 2PI graphs with degree $\omega \leq n$.

- It follows that $G(k_3 = k_3^*, k_0)$ is directly related to $G_{2PI}(\bar{k}_3, k_0)$ with $\bar{k}_3$ chosen such that $\langle N_3 \rangle_{G_{2PI}} = 1/2$.
- In particular

$$\left. \frac{\langle \omega \rangle}{\langle N_3 \rangle} \right|_{k_3 = k_3^*} = \left. \frac{\langle \omega \rangle_{2PI}}{\langle N_3 \rangle_{2PI}} \right|_{\langle N_3 \rangle_{2PI} = 1/2} = 2 \left. \langle \omega \rangle_{2PI} \right|_{\langle N_3 \rangle_{2PI} = 1/2}$$

- To find $\langle \omega \rangle_{2PI}$ up to order $(e^{-k_0})^n$ we only need the 2PI graphs with degree $\omega \leq n$.
- Here the coloring is useful, and the relation to quadrangulations of Riemann surfaces.



Conclusions & Outlook

- Conclusions
  - There seems to be a closer connection between colored tensor models and colored DT than between standard tensor models and uncolored DT.
  - Monte Carlo simulations suggest that colored DT and uncolored DT sit in the same universality class.
  - The simple representation of triangulations through colored graphs may open up opportunities for analytical calculations.

Thanks to Biancha Dittrich, Razvan Gurau and Valentin Bonzom for suggesting a project along these lines!

Conclusions & Outlook

- Conclusions
  - There seems to be a closer connection between colored tensor models and colored DT than between standard tensor models and uncolored DT.
  - Monte Carlo simulations suggest that colored DT and uncolored DT sit in the same universality class.
  - The simple representation of triangulations through colored graphs may open up opportunities for analytical calculations.
- Outlook
  - General algorithm to construct all low-ω graphs?
  - Colored DT with measure term? $Z = \sum_{\mathcal{G}} \frac{1}{C_{\mathcal{G}}} \left(\prod_e d_e^{\beta}\right) e^{-S_{\mathrm{DT}}}$

Thanks to Biancha Dittrich, Razvan Gurau and Valentin Bonzom for suggesting a project along these lines!