EFFECTIVE FIELD THEORIES in PARTICLE PHYSICS

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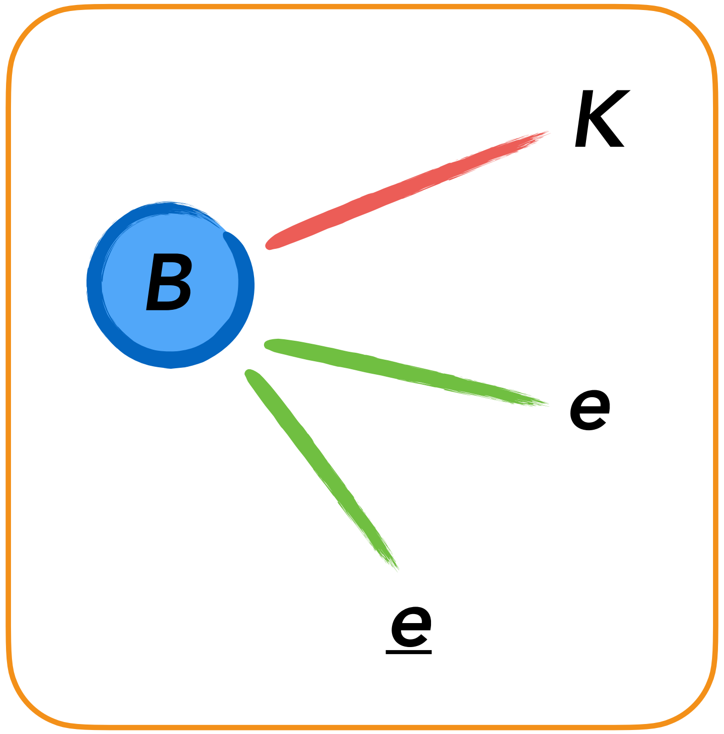
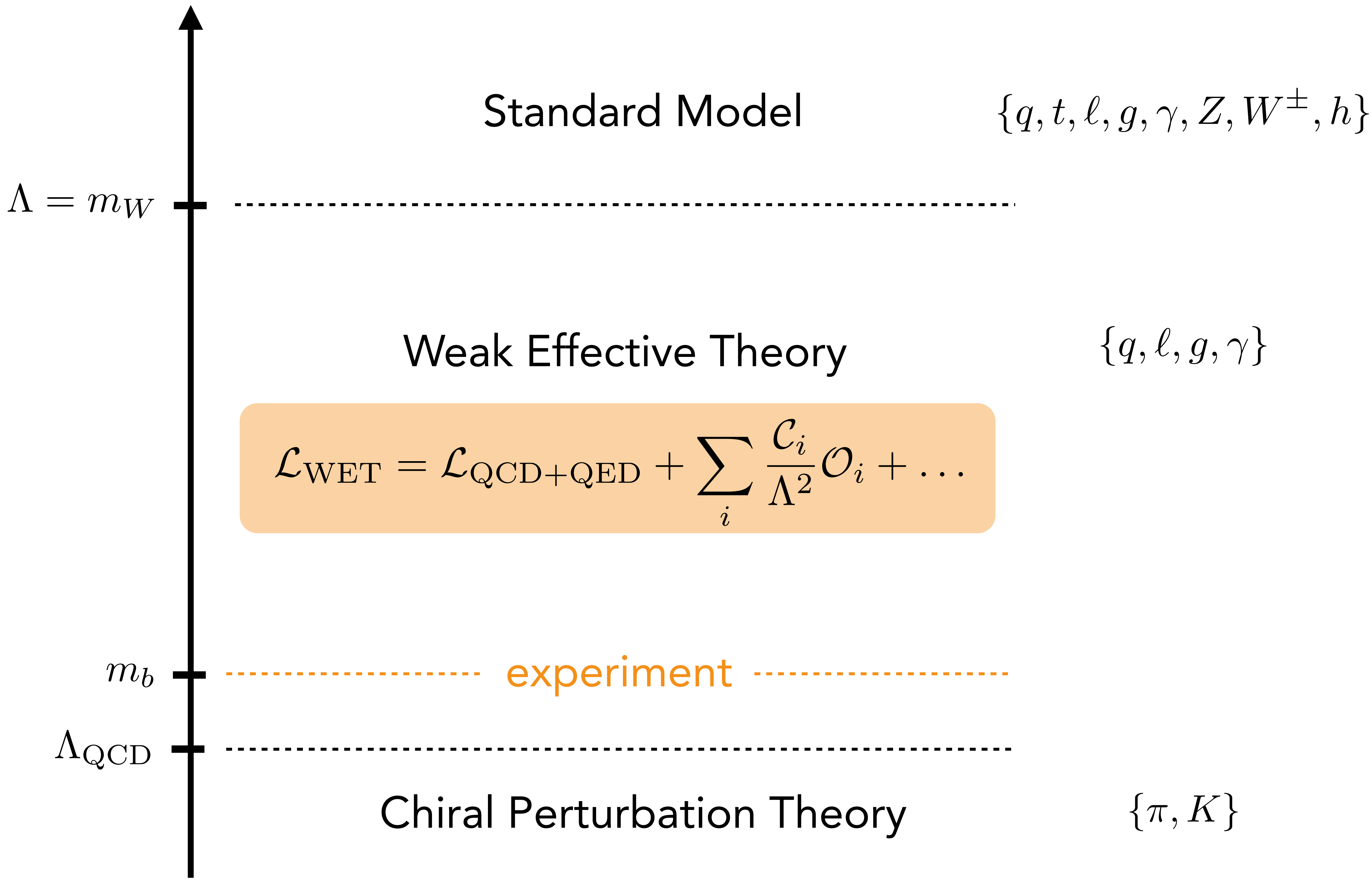
Part II

Weak Effective Theory



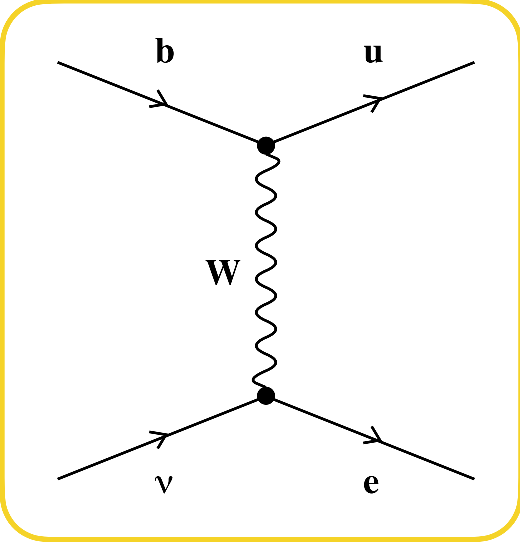
Multi-scale physics



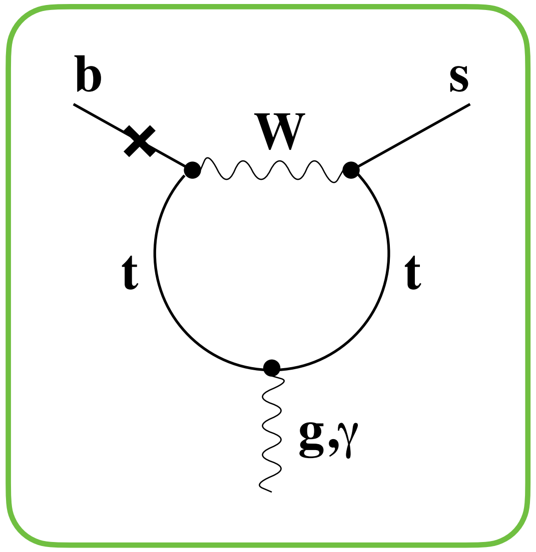
Weak Effective Theory

Goal: precise description of flavor-changing processes.

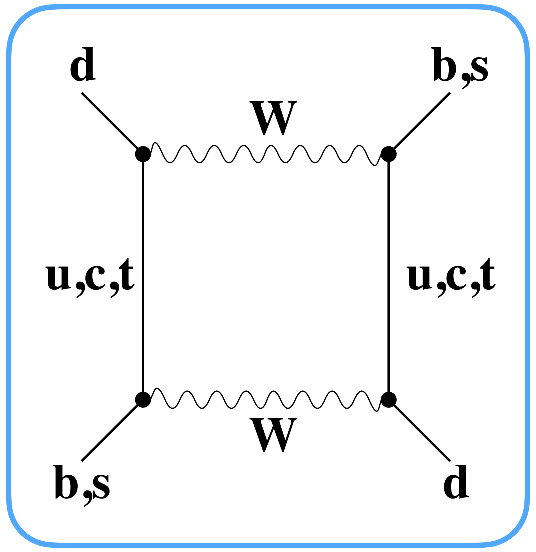
$\overline{B}^0 \to \pi^+ e^- \bar{\nu}_e$



$B \to X_s \gamma$



$B \leftrightarrow \overline{B}$



$$\mathcal{L}_{\text{WET}} = -\frac{4G_F}{\sqrt{2}} \sum_i C_i\, O_i^{(6)} + \mathcal{O}\left(\frac{m_b^2}{m_W^2}\right)$$

What are the relevant operators?

introduction: Buras 1998

Matching

$$\mathcal{M} = \boxed{\langle f|\mathcal{L}_{\mathrm{SM}}|i\rangle \stackrel{!}{=} -\frac{4G_F}{\sqrt{2}}\sum_i C_i(\mu)\langle f|O_i^{(6)}(\mu)|i\rangle}$$

- match full SM amplitude onto weak effective theory
- include all operators that respect the SM symmetries

Matching

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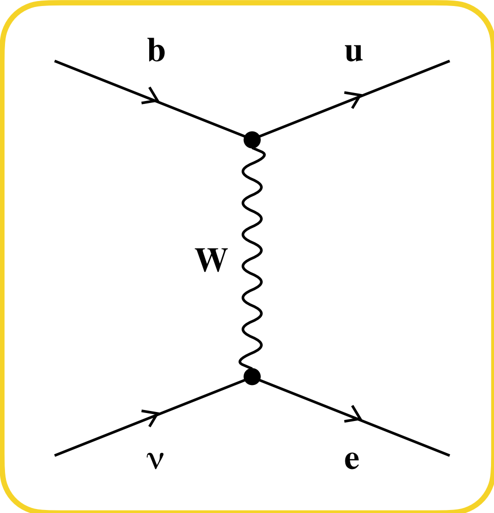
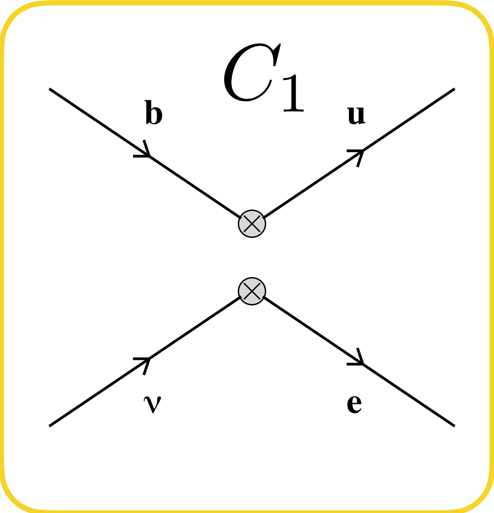


match



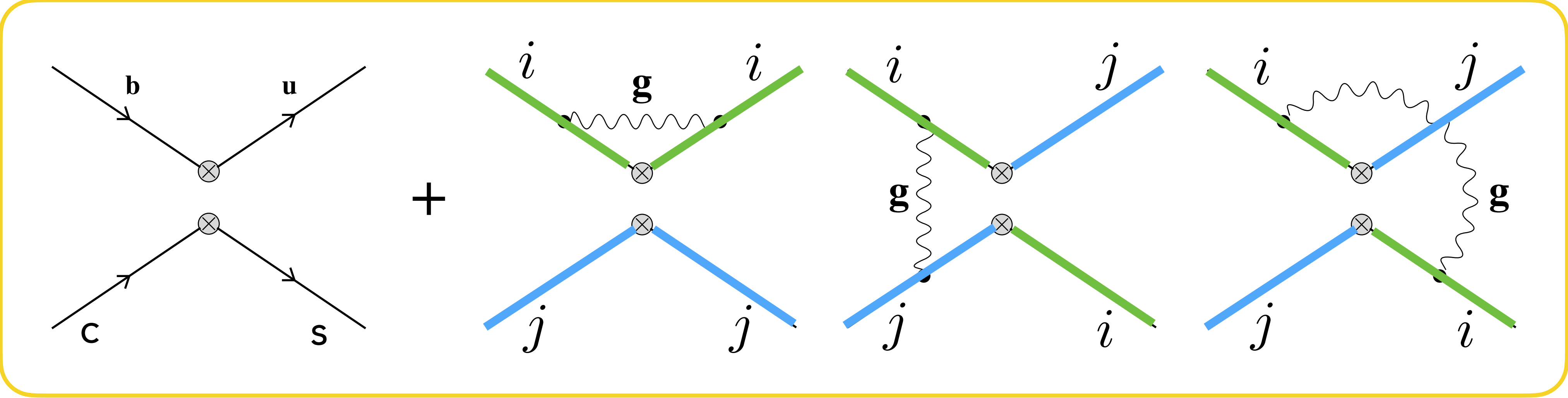
$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} C_1 O_1$$

$$O_1 = V_{ub}(\bar{u}_L\gamma^\mu b_L)(\bar{e}_L\gamma_\mu\nu_L)$$

$$C_1 = 1$$

Matching with QCD effects

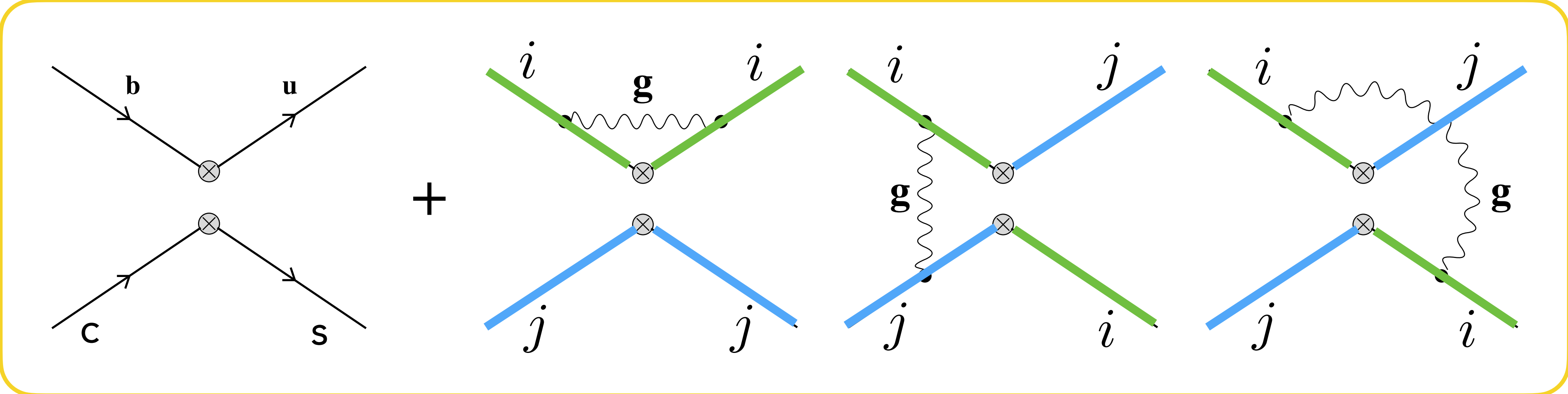
$\overline{B}^0 \to \pi^+ D_s^-$



$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ub} \left[C_1(\mu)\, \bar{s}_L^j \gamma_\mu c_L^j\, \bar{u}_L^i \gamma^\mu b_L^i + C_2(\mu)\, \bar{s}_L^i \gamma_\mu c_L^j\, \bar{u}_L^j \gamma^\mu b_L^i\right]$$

Matching with QCD effects

$\overline{B}^0 \to \pi^+ D_s^-$



$$\mathcal{L}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ub} \left[C_1(\mu)\, \bar{s}_L^j \gamma_\mu c_L^j\, \bar{u}_L^i \gamma^\mu b_L^i + C_2(\mu)\, \bar{s}_L^i \gamma_\mu c_L^j\, \bar{u}_L^j \gamma^\mu b_L^i\right]$$

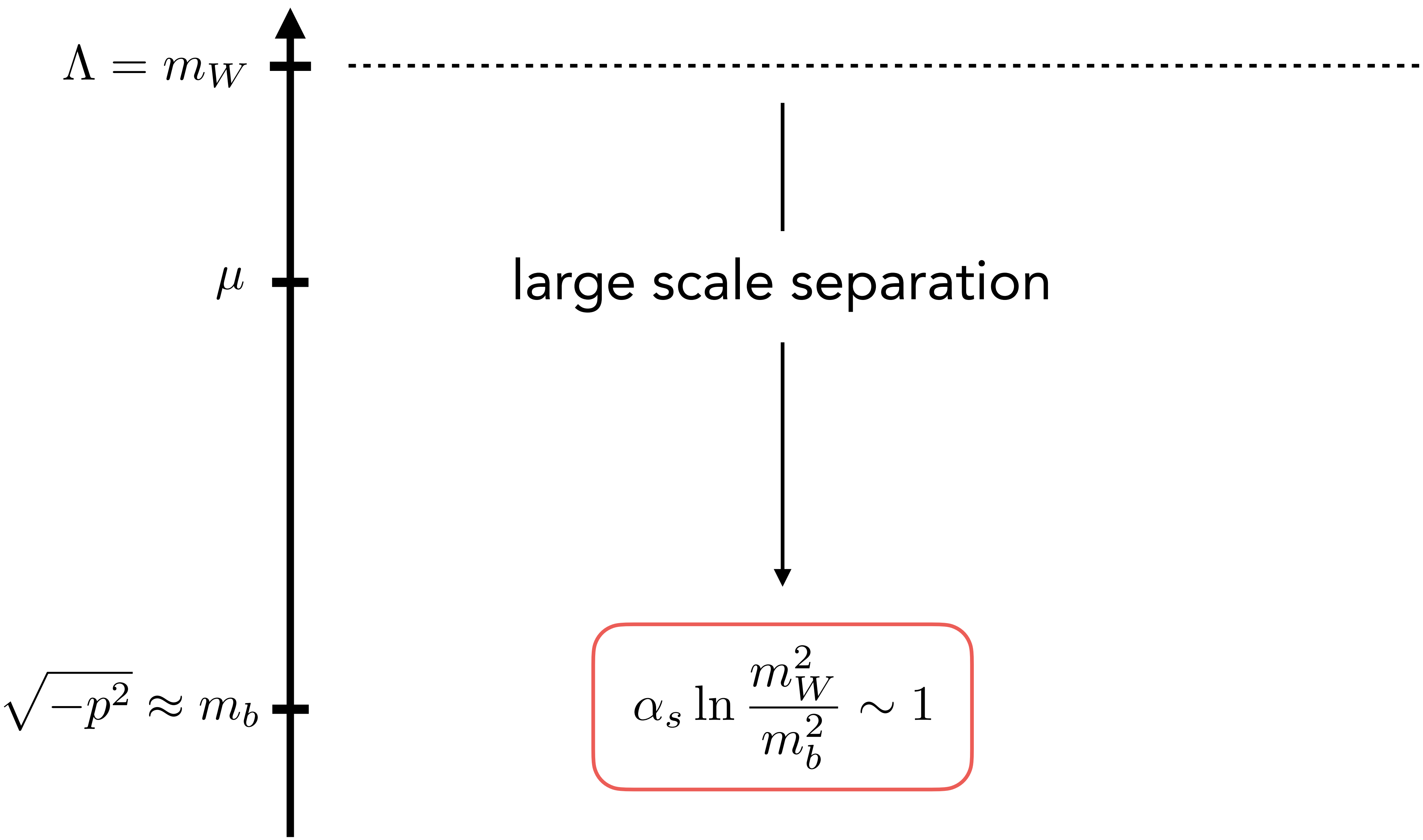
- Wilson coefficients depend on matching scale:

$$C_1(\mu) = 1 + \frac{3}{N_c}\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)$$

$$C_2(\mu) = -3\frac{\alpha_s(\mu)}{4\pi}\left(\ln\frac{M_W^2}{\mu^2} - \frac{11}{6}\right) + O(\alpha_s^2)$$

Running couplings

- factorize amplitude: $\underbrace{1+\alpha_s \ln \frac{M_W^2}{-p^2}}_{\text{full theory}} = \underbrace{\left(1+\alpha_s \ln \frac{M_W^2}{\mu^2}\right)}_{C(\mu)} \underbrace{\left(1+\alpha_s \ln \frac{\mu^2}{-p^2}\right)}_{\langle O(\mu)\rangle} + \ldots$



Running couplings

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$\Lambda = m_W$

$\mu \lesssim m_W$ — match amplitudes $\longrightarrow$ $C(\mu_W)$

$\downarrow$ renormalization group evolution resums $\left(\alpha_s \ln\frac{m_W^2}{m_b^2}\right)^n$

$\sqrt{-p^2} \approx m_b$ — $\mathcal{M}_{\text{eff}} \sim C(m_b)\langle O(m_b)\rangle$ $\longleftarrow$ $C(m_b) = U(m_b,\mu_W)C(\mu_W)$

Running couplings - example

$$\overline{B}^0 \to \pi^+ D_s^-$$

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} V_{cs}^* V_{ub} \left[C_1(\mu)\, \bar{s}_L^j \gamma_\mu c_L^j \, \bar{u}_L^i \gamma^\mu b_L^i + C_2(\mu)\, \bar{s}_L^i \gamma_\mu c_L^j \, \bar{u}_L^j \gamma^\mu b_L^i \right]$$

$$C_1(m_W) = 1 + \mathcal{O}(\alpha_s)$$

$$C_2(m_W) = 0 + \mathcal{O}(\alpha_s)$$

match & run →

$$C_1(m_b) \approx 1.1$$

$$C_2(m_b) \approx -0.2$$

Effect on decay branching ratio $\mathcal{B}(\overline{B}^0 \to \pi^+ D_s^-) \sim C_i^2$.

Buchalla, Buras, Lautenbacher 1995

Summary Part II

Weak Effective Theory:

- good description of charged currents at $E \ll m_W$
- effective Lagrangian:

$$\mathcal{L}_{\mathrm{WET}} = -\frac{4G_F}{\sqrt{2}} \sum_i C_i \, O_i^{(6)} + \mathcal{O}\left(\frac{m_b^2}{m_W^2}\right)$$

Obtain *B*-physics amplitude $\mathcal{M}_{\mathrm{eff}} \sim C(m_b)\langle O(m_b)\rangle$:

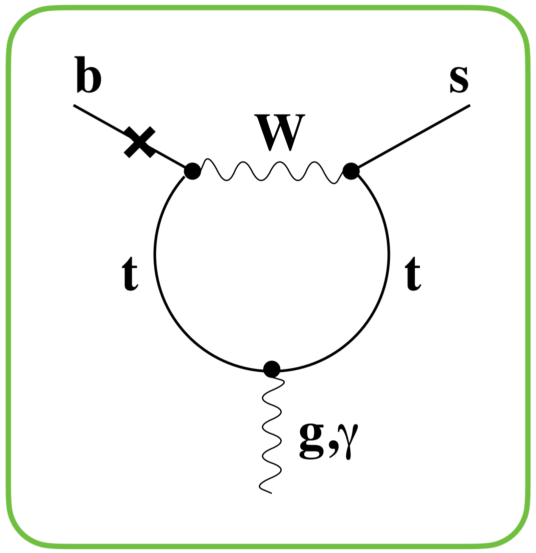
- match $\longrightarrow$ $C(\mu_W)$ (flavor-universal)
- evolve $\longrightarrow$ $C(m_b) = U(m_b, \mu_W) C(\mu_W)$

WET makes more precise predictions than full theory.*

*at the same perturbative order

Your turn

$B \to X_s \gamma$

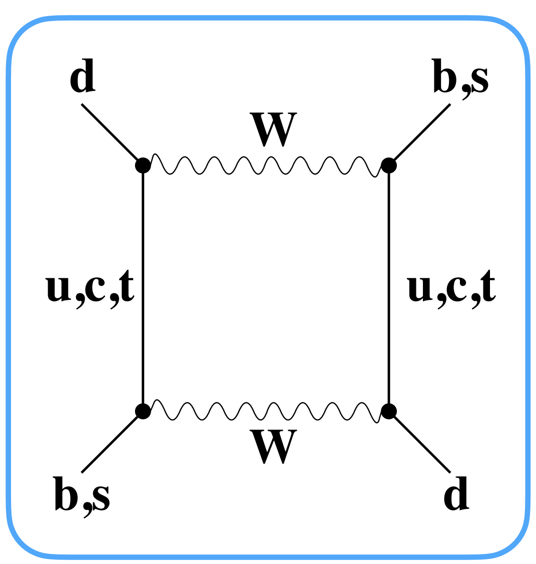


$$Q_{7\gamma} = \frac{e}{8\pi^2} m_{\rm b} \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) b_i F_{\mu\nu}$$

$$Q_{8G} = \frac{g}{8\pi^2} m_{\rm b} \bar{s}_i \sigma^{\mu\nu} (1+\gamma_5) T^a_{ij} b_j G^a_{\mu\nu}$$

- Draw Feynman diagrams of the two operators.
- Why does Q_{8G} contribute to $B \to X_s \gamma$?

$B \leftrightarrow \overline{B}$



- Which WET operators contribute?