QCD

at
e^+e^- Colliders

(LEP + SLC)

W. J. Metzger
University of Nijmegen

NNV, Petten
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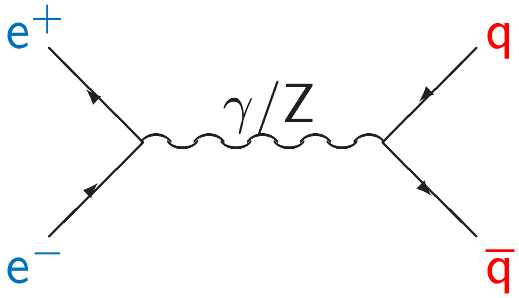
History

1897 Discovery of the electron
The first elementary particle
Birth of Particle Physics

1947 Discovery of the pion
Begin to understand Strong Interaction

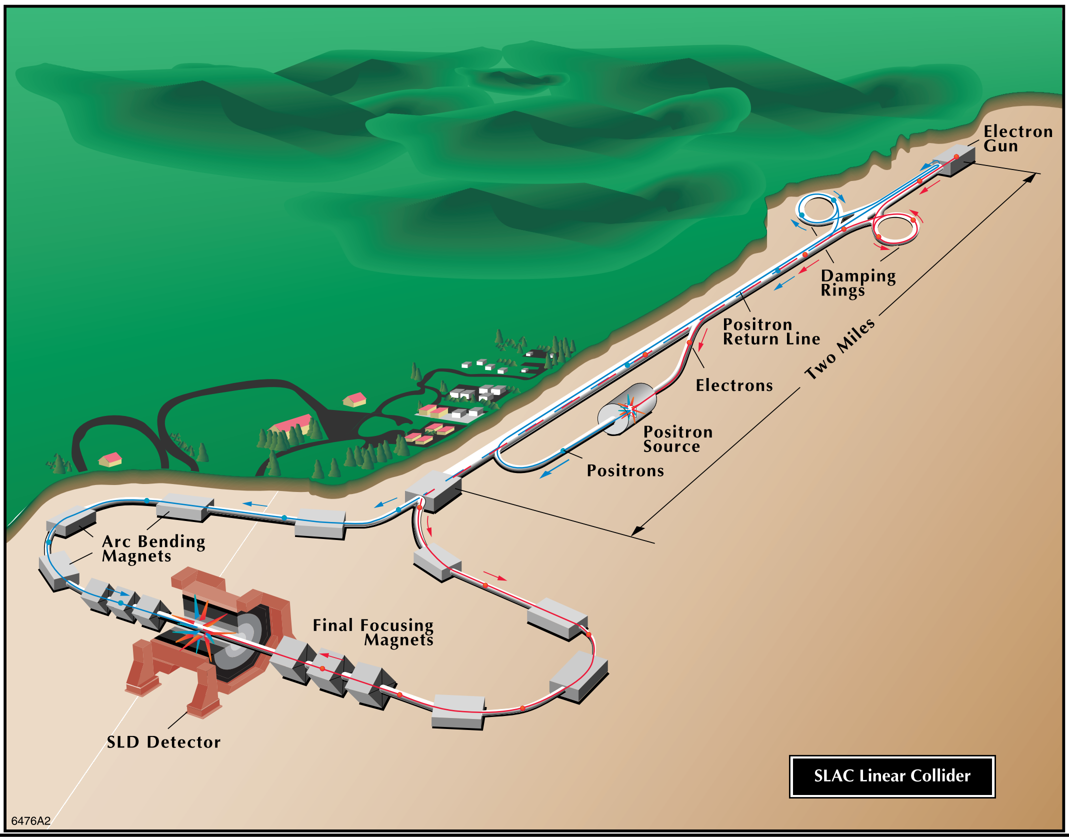
1997 Still trying to understand Strong Interaction
now with the theory of QCD
and with the help of electrons (and other particles)

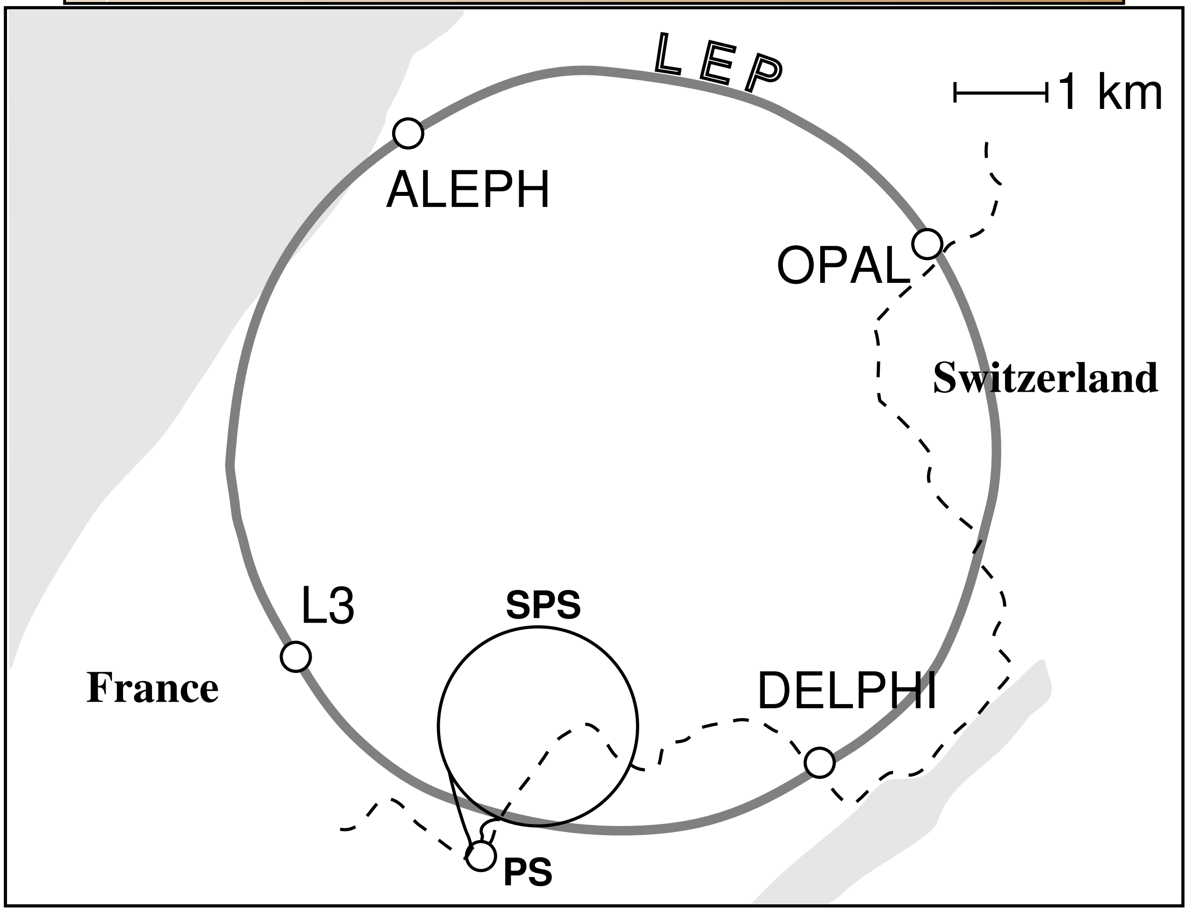


- Well defined production process
  leading to a clean final state

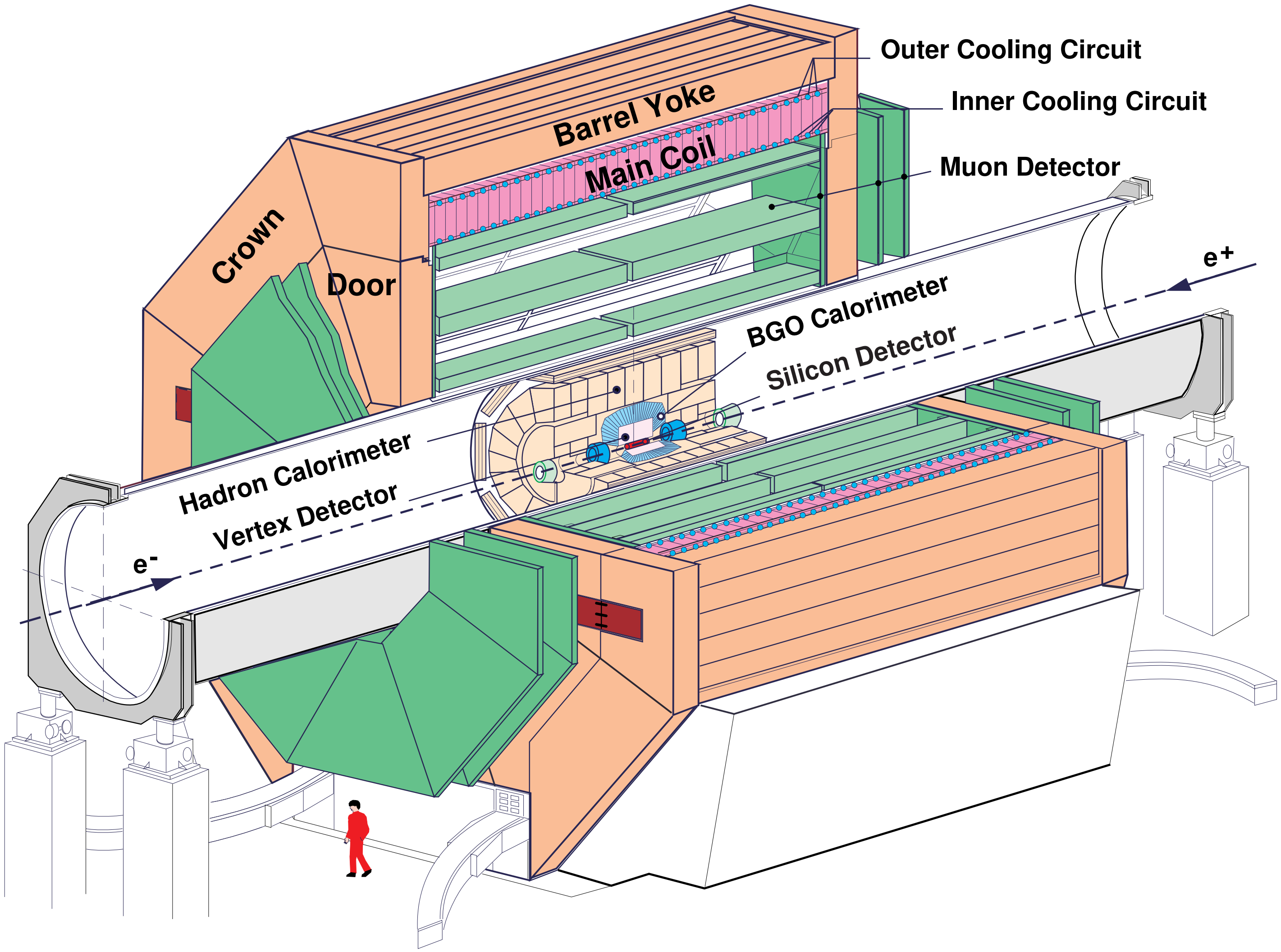
- Can extract physics with a minimum of assumptions

Obligatory Orientation – The Machines





Obligatory Orientation – A "Typical" Detector

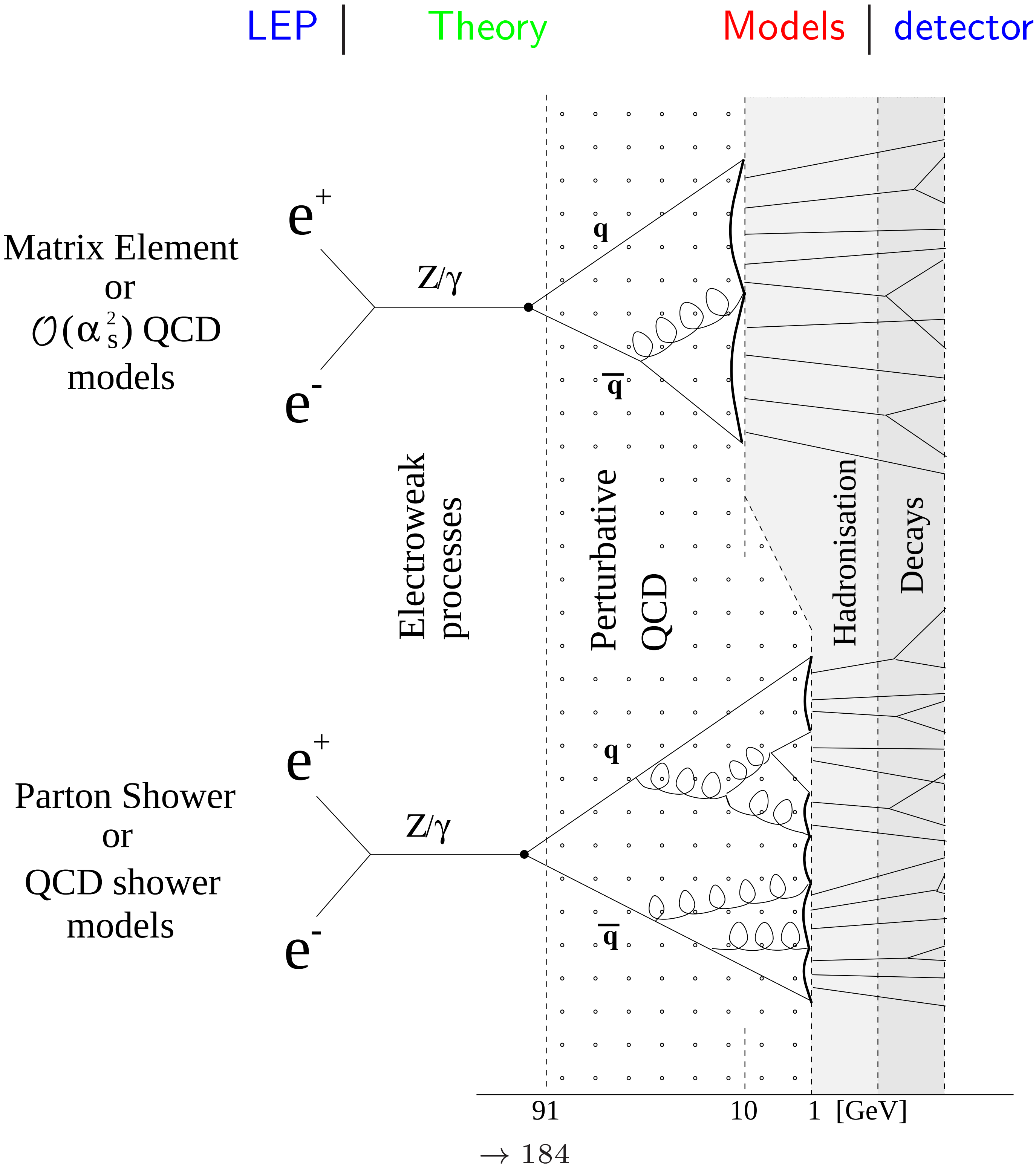


L3

Outline

- Introduction
- QCD – some basic tests
  - Does g have spin 1 ?
  - Is SU(3) the colour group ?
  - Does α_s run ?
- Other perturbative QCD results
  - Gluon Splitting
  - Energy dependence of parton shower
- Investigating softer aspects of QCD
  - quark/gluon fragmentation differences
  - angular intermittancy
  - subjets
  - multiplicities
- Conclusions

$\color{red}{Q}\color{green}{C}\color{blue}{D}$ in e^+e^- collisions at LEP

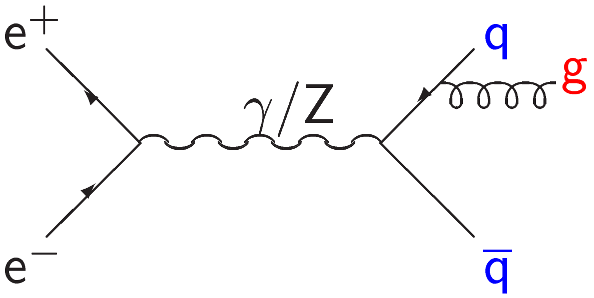


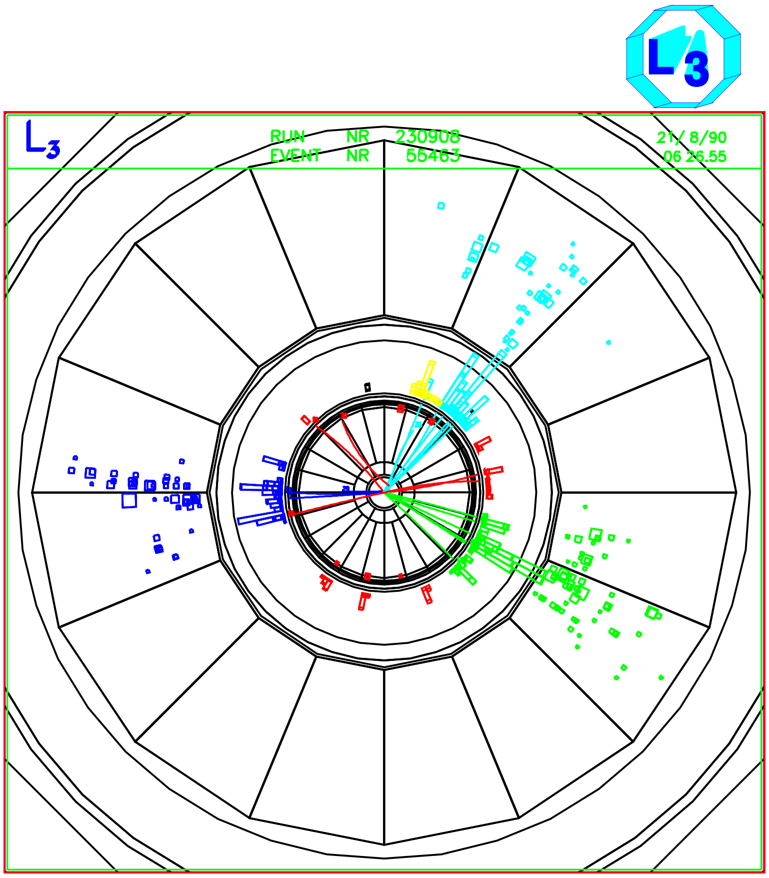
QCD

- $e^+e^- \rightarrow q\bar{q}$ (Electro-Weak)
  $\rightarrow$ 2 jets (~kinematics)

- QCD: $q \rightarrow qg \rightarrow$ 3-jet events
  (also 4-, 5-, ... jet events)

  seen long ago at PETRA (Germany not Jordan)
  (not that long ago)

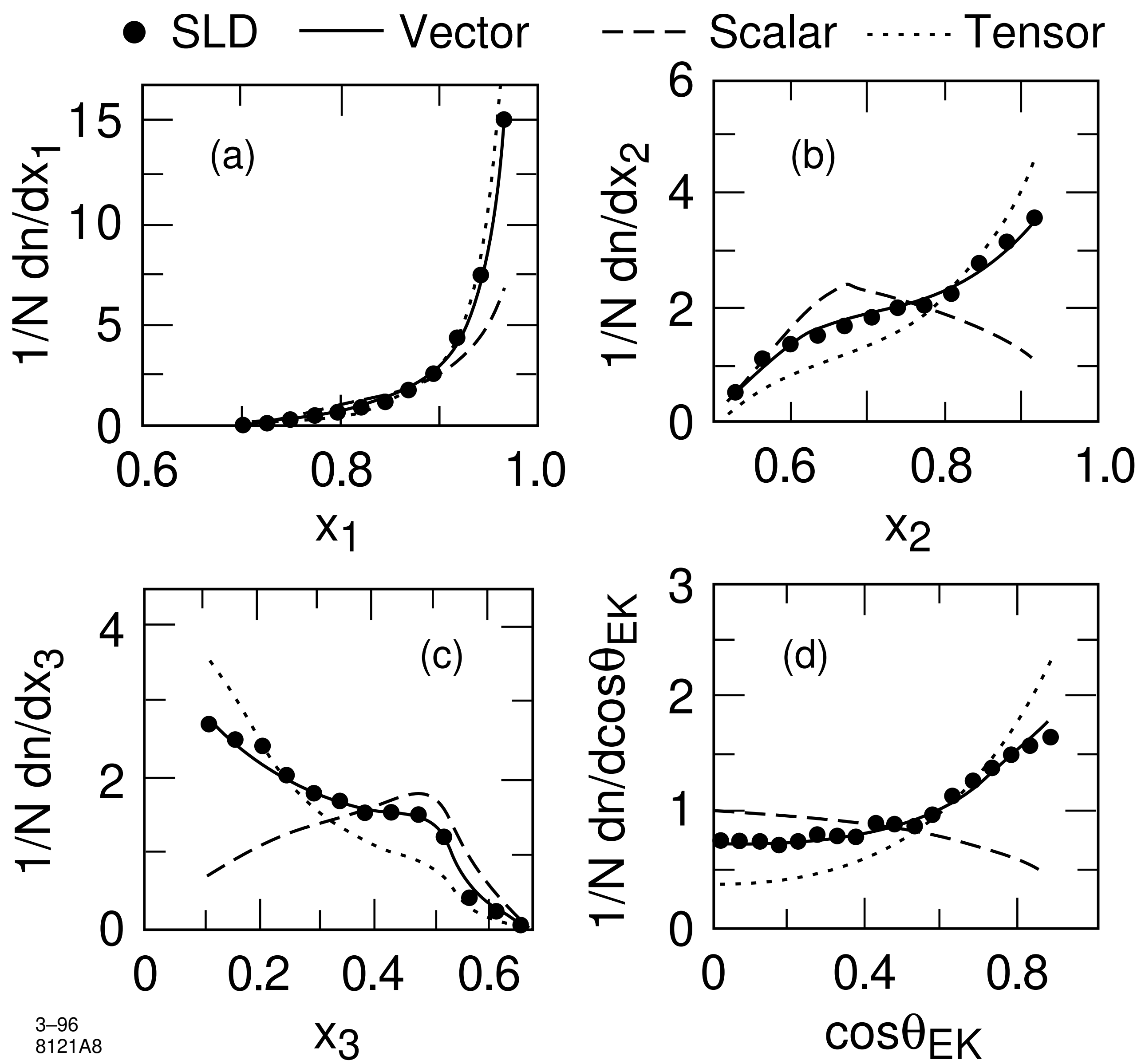




Does g have spin 1 ?

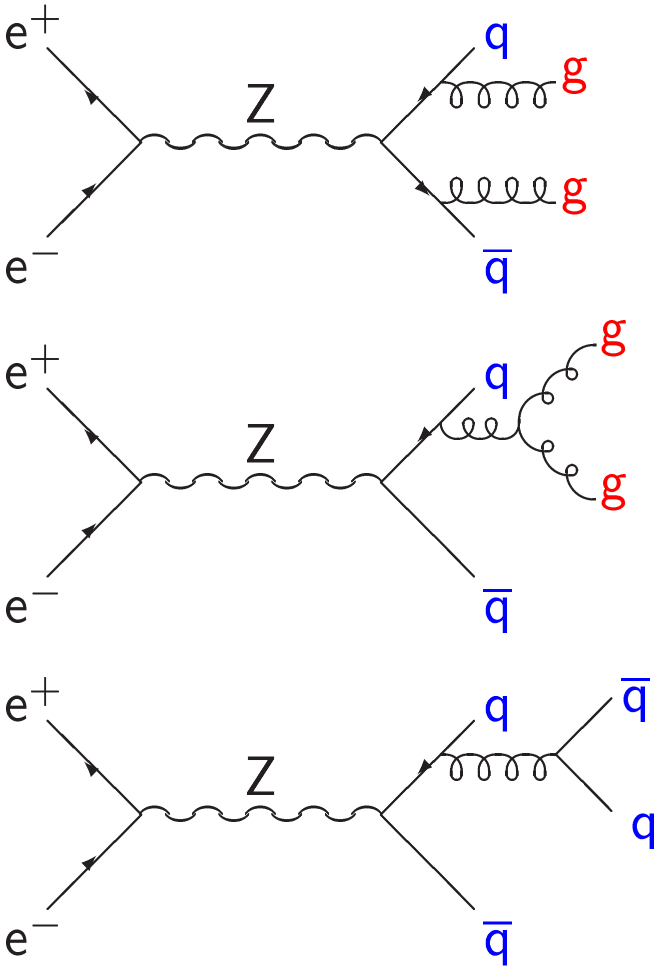
Energy and Ang. dists. of 3-jet events:
$x_i = \frac{2E_i}{E_{\text{cms}}}$, $E_1 > E_2 > E_3$
θ_{EK} = angle between jets 2,3 in rest frame of 1,2

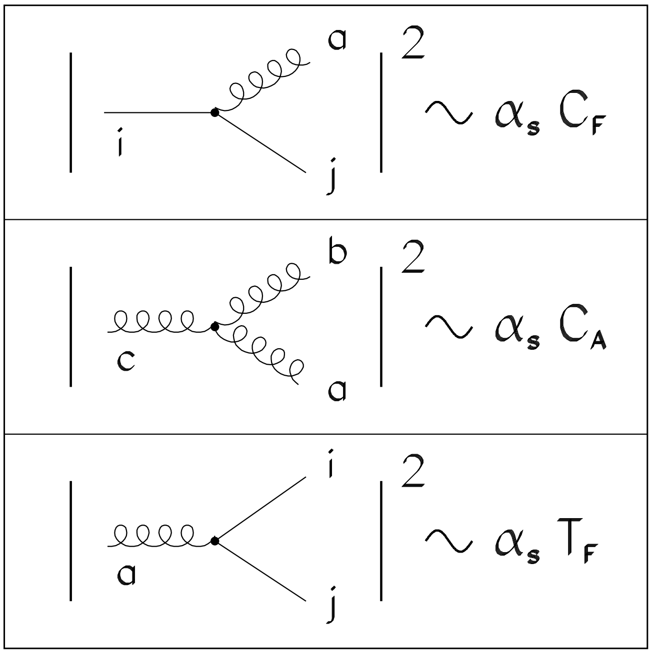
- Not spin 0, consistent with spin 1 – PETRA
- Not spin 2 – SLD: (PRD 55(1997)2533)



Is SU(3) the colour group ?

ALEPH (CERN-PPE/97-002): in 4-jet events,



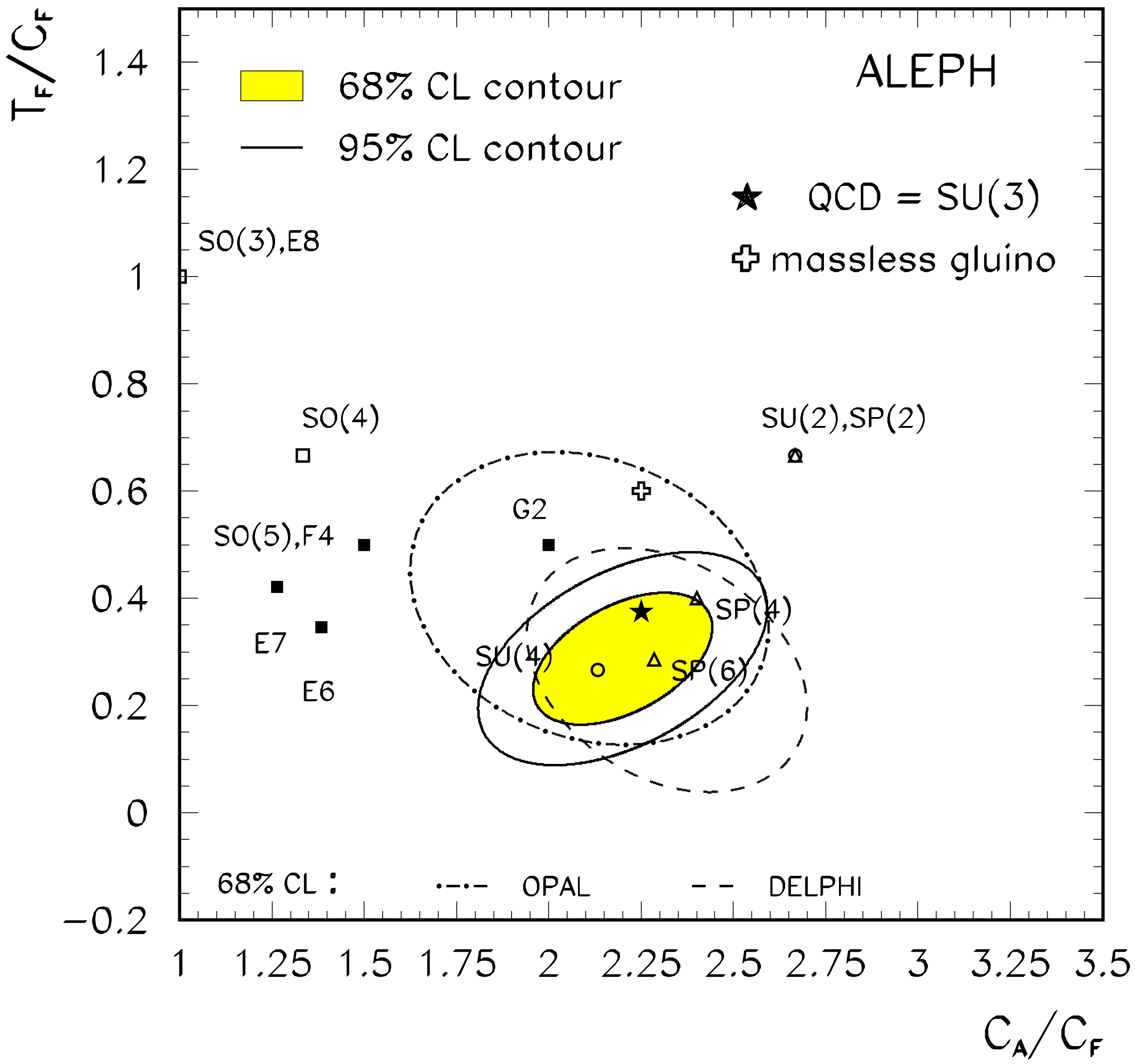


4-jet diagrams are related to the Casimir operators (colour factors) of colour group of QCD: C_{F}, C_{A}, T_{F}

From relative frequencies of 4-jet diagrams, can measure C_{F}, C_{A}, T_{F}

Jet E and Ang. dists. of diagrams differ

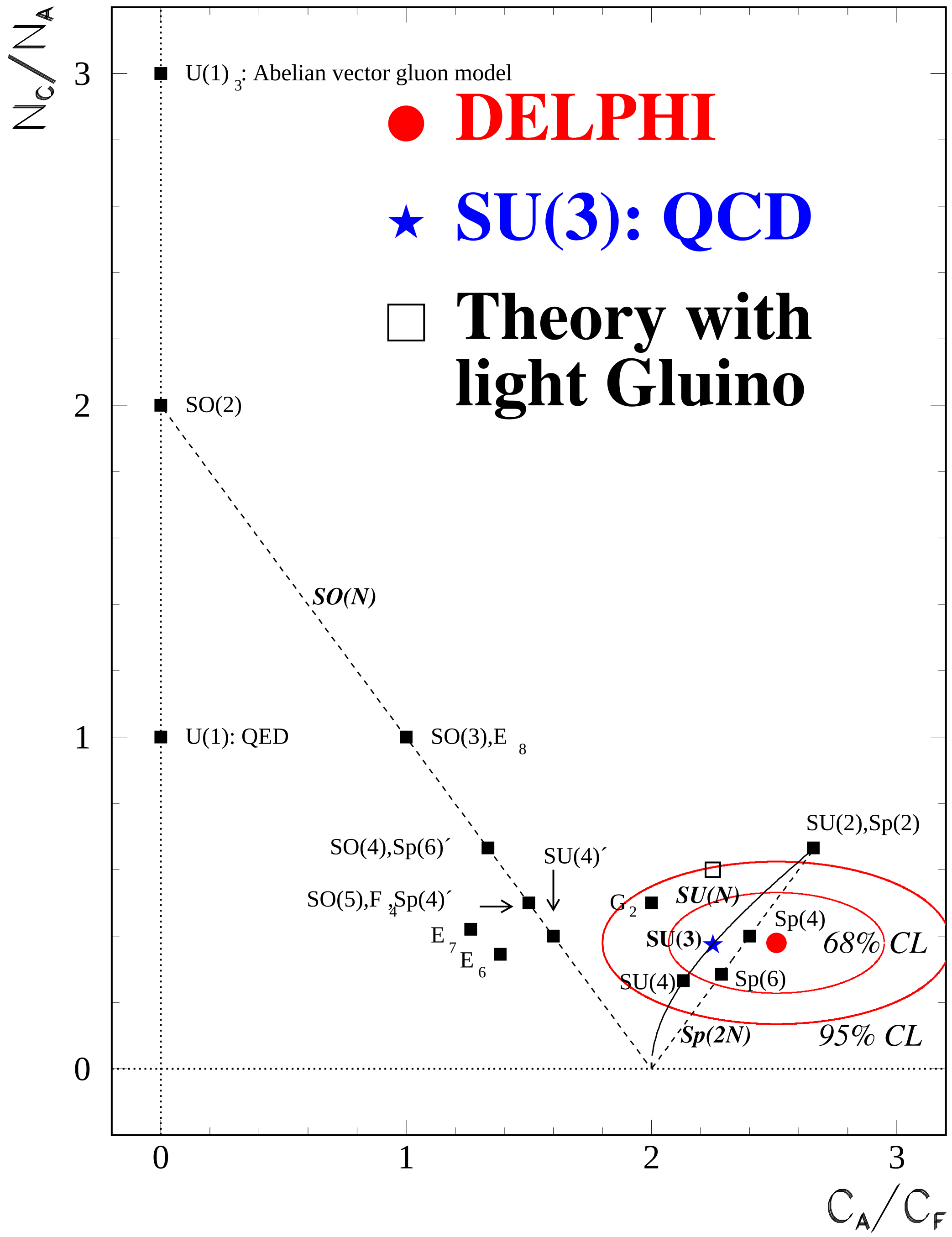
Colour Factors



Results agree with SU(3)

Colour Factors

DELPHI has done a (new) similar analysis (EPS97 236):



Results agree with SU(3)
(and with ALEPH, but errors are larger)

Does α_s run ?

- Measure α_s at different scales Q^2
- Check if $\alpha_s(Q^2)$ depends on Q^2 as predicted by QCD

$$\alpha_s(Q^2) = \frac{4\pi}{\beta_0 \ln(Q^2/\Lambda^2)} \left\{ 1 - \frac{2\beta_1}{\beta_0^2} \frac{\ln(\ln(Q^2/\Lambda^2))}{\ln(Q^2/\Lambda^2)} + \ldots \right\}$$

$$\beta_0 = 11 - \frac{2}{3} n_f \qquad \beta_1 = 102 - \frac{38}{3} n_f$$

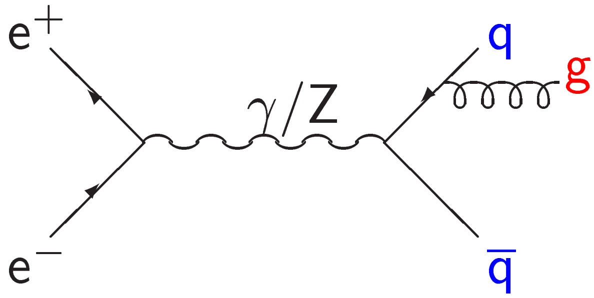
where n_f = the number of flavours

N.B. n_f depends on available E
– at low E, heavy quarks do not contribute

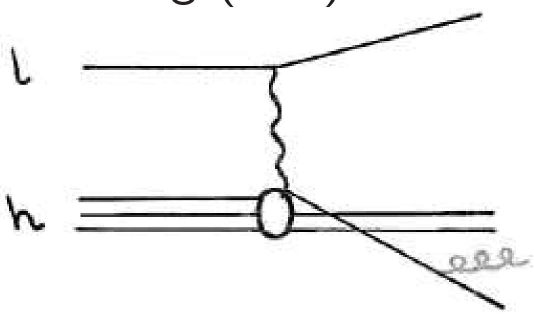
Measuring α_s

Can measure α_s practically anywhere you have hadrons

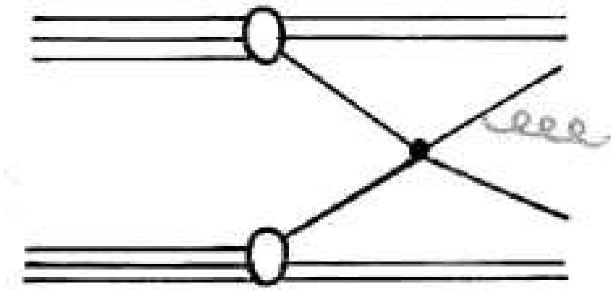
- e^+e^-



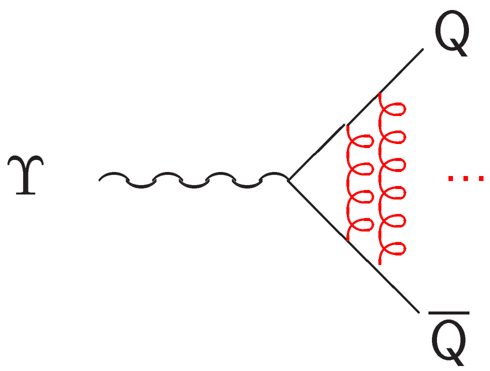
- deep inelastic scattering (DIS)



- hadron-hadron (hh)



- heavy quarkonia



Measuring α_s

- Choose an OBSERVABLE f
  - IR- and collinear-safe

- Choose RENORMALISATION SCHEME
  - generally $\overline{\text{MS}}$

- Choose RENORMALISATION SCALE (μ^2)
  - to minimize importance of higher-order terms
  - no consensus, much controversy

- Calculate in PERTURBATION THEORY
  - at least to NLO:

    e^+e^-, ℓh: $\mathcal{O}(\alpha_s^2)$ hh: $\mathcal{O}(\alpha_s^3)$

  $\Longrightarrow f(\mu^2, \Lambda_{\overline{\text{MS}}})$

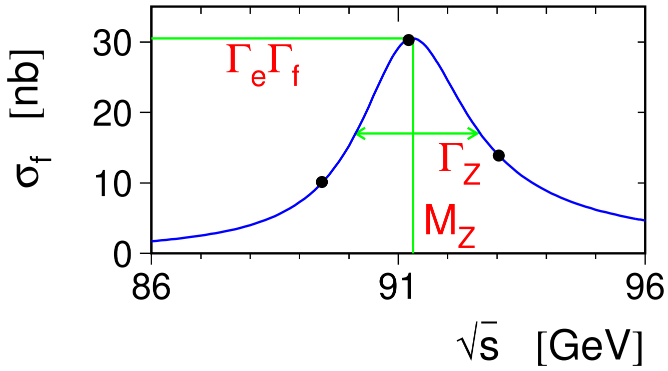
- Measure the OBSERVABLE at scale μ^2

- measurement + calculation $\Longrightarrow \Lambda_{\overline{\text{MS}}}$

  With $\Lambda_{\overline{\text{MS}}}$ calculate $\alpha_s(Q^2)$ for any Q^2

Measuring α_s in e^+e^-

- Z lineshape



$$\Gamma_Z = \Gamma_{had} + \Gamma_{ee} + \Gamma_{\mu\mu} + \Gamma_{\tau\tau} + 3\Gamma_{\nu\nu}$$

$$\Gamma_{had} = 1.671\left(1 + \left(\frac{\alpha_s}{\pi}\right) + 0.75\left(\frac{\alpha_s}{\pi}\right)^2 - 15.3\left(\frac{\alpha_s}{\pi}\right)^3 + \ldots\right)$$

Concern about convergence of series
$\mathcal{O}(\alpha_s^4)$ calculation in progress
Applicable at only 1 scale

- τ decays

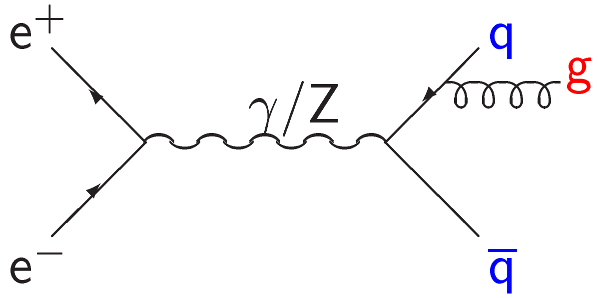
$$R_\tau = \frac{\mathrm{BR}(\tau \to \mathrm{had.} + \nu_\tau)}{\mathrm{BR}(e\overline{\nu}_e\nu_\tau)} = 3.058\left(1 + \delta_{\mathrm{pert.}}(\alpha_s) + \delta_{\mathrm{non-pert.}}(\alpha_s)\right)$$

Large theoretical uncertainty since M_τ small
Applicable at only 1 scale

Measuring α_s in e^+e^-

- 3-jet fraction

  dependence on jet algorithm
  scale uncertainties
  M.E. only to $\mathcal{O}(\alpha_s^2)$



- Hadronic event shapes (see later)
  - the more gluons radiated, the less 2-jet-like

- Scaling Violatons in fragmentation functions
  Choice of scale particularly difficult (see later)

To check running of α_s, use same method at various scales to reduce systematic errors.

Use hadronic event shapes at various E_{cms}

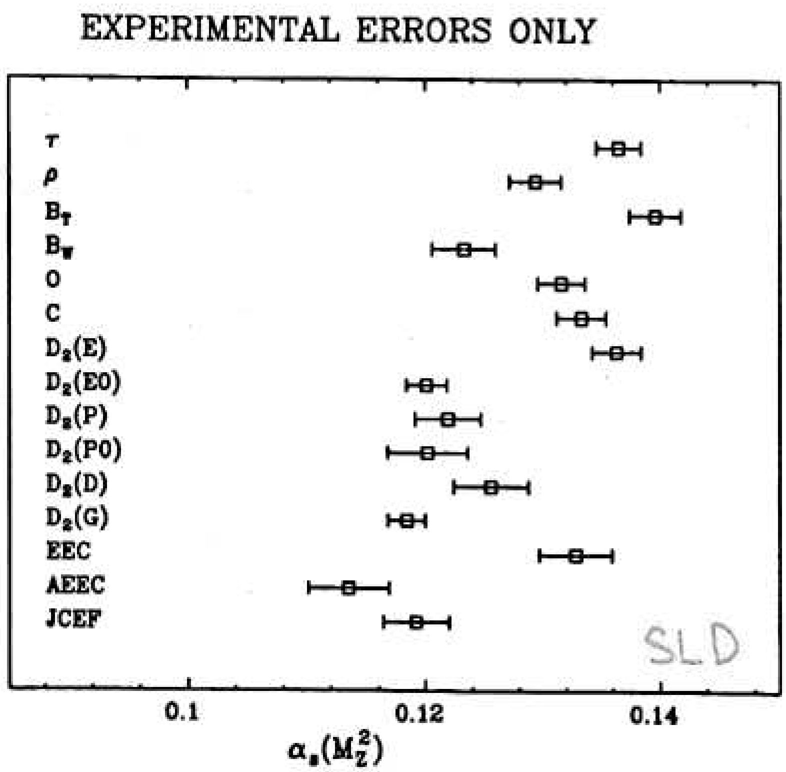
α_s from shape variables, I

Measure dist., $R(\alpha_s, Y) \equiv \frac{1}{\sigma}\frac{d\sigma}{dY}$, of shape variable, Y,
From $\mathcal{O}(\alpha_s^2)$ matrix elements, R is given by

$$R(\alpha_s, Y) = \frac{\alpha_s}{2\pi} A(Y) + \left(\frac{\alpha_s}{2\pi}\right)^2 \left[B(Y) + 2\pi\beta_0 \ln\frac{\mu^2}{s} A(Y)\right]$$

So fit this to measured $R \Longrightarrow \alpha_s$
But α_s from this expression is disastrous:

Fix renormalisation scale: $\mu^2 = s$



PRD 51 (1995) 962

- 15 highly correlated observables measured using same data sample yield highly discrepant values of $\alpha_s(M_Z^2)$
- Escape clause for QCD: higher-order terms must be important

α_s from shape variables, II

- Relative importance of $\mathcal{O}(\alpha_s^2)$ and $\mathcal{O}(\alpha_s)$ terms is nearly 1 for some shape variables.

- This suggests using different values of μ for different shape variables.

- But what? Theorists disagree on best 'optimized scale'.

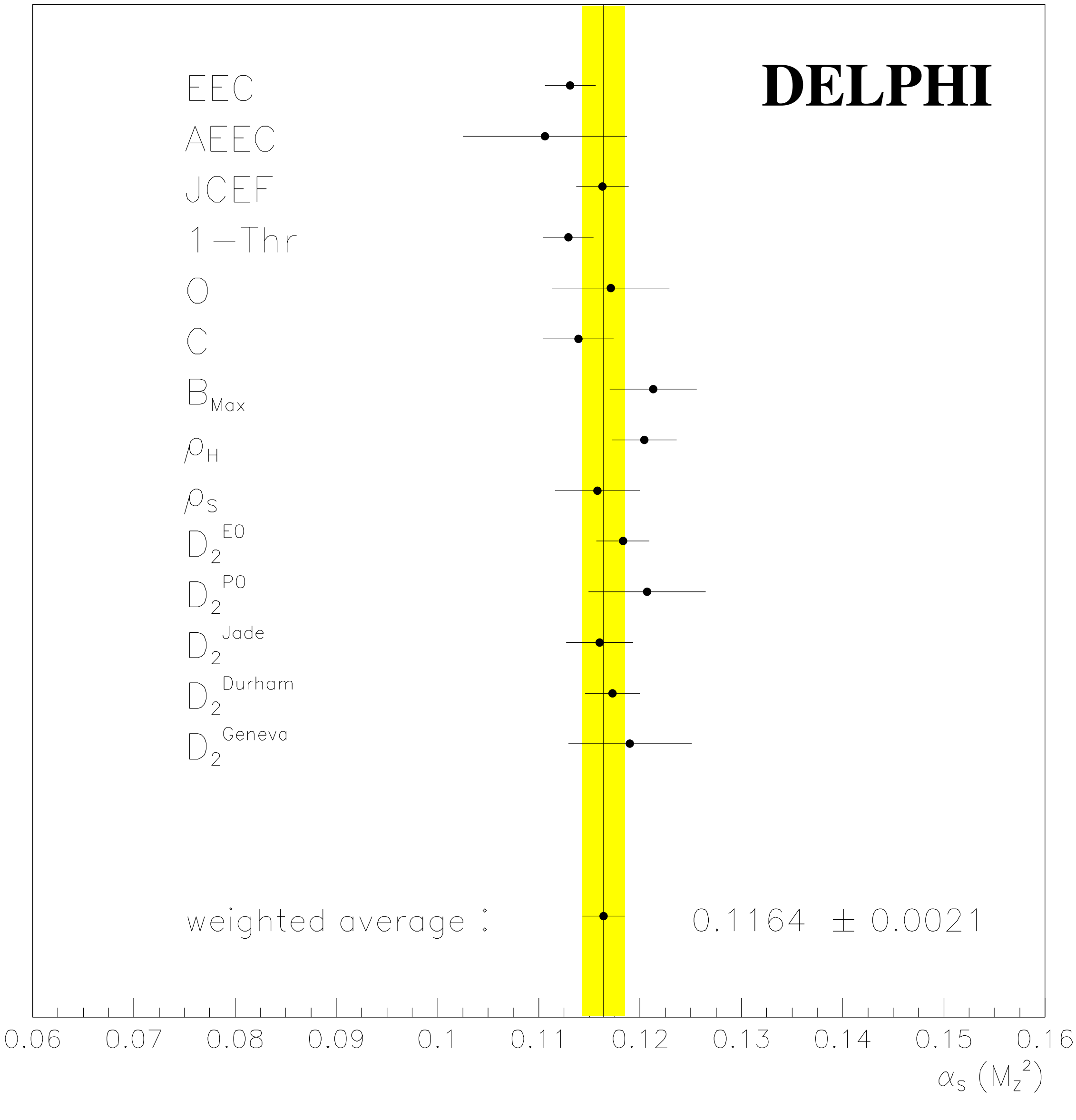
- DELPHI (EPS97 543) (1.5M Z events)
  "Let the data decide:" Use $\mu^2 = x_\mu s$,
  x_μ an additional fit parameter instead of $x_\mu = 1$

- and fit to $R_\mu(\alpha_s, x_\mu, Y) \equiv \frac{1}{\sigma}\frac{d^2\sigma}{dY\, d\cos\theta_T}$

Observable	Fit Range (Obs.)	Fit range ($\cos\vartheta_T$)	x_μ	χ^2/n_{df}
$1-T$	0.05 - 0.30	0.0 - 0.84	0.0034 ± 0.0002	1.24
O	0.24 - 0.44	0.0 - 0.84	1.76 ± 0.42	0.69
C	0.28 - 0.72	0.0 - 0.84	0.0047 ± 0.0004	1.23
B_{Max}	0.12 - 0.24	0.0 - 0.72	0.027 ± 0.014	0.96
ρ_H	0.03 - 0.30	0.0 - 0.84	0.0726 ± 0.0034	1.13
ρ_S	0.04 - 0.20	0.0 - 0.84	0.0104 ± 0.0006	1.01
ρ_D	0.01 - 0.30	0.0 - 0.84	1.15 ± 0.02	1.02
D_2^{E0}	0.05 - 0.20	0.0 - 0.84	0.113 ± 0.023	0.80
D_2^{P0}	0.05 - 0.20	0.0 - 0.72	0.075 ± 0.032	0.88
D_2^{Jade}	0.05 - 0.25	0.0 - 0.84	0.073 ± 0.014	0.85
D_2^{Durham}	0.02 - 0.12	0.0 - 0.84	0.0082 ± 0.0016	0.97
D_2^{Geneva}	0.07 - 0.14	0.0 - 0.84	0.11 ± 1.91	1.32
EEC	$28.8^\circ - 158.4^\circ$	0.0 - 0.84	0.0094 ± 0.0004	1.00
$AEEC$	$25.2^\circ - 75.6^\circ$	0.0 - 0.48	0.009 ± 0.028	0.70
$JCEF$	$104.4^\circ - 169.2^\circ$	0.0 - 0.84	0.0987 ± 0.0068	1.02

Large differences in x_μ and $x_\mu \neq 1$ $\Uparrow$

α_s from shape variables, II



Consistent values of α_s

α_s from shape variables, III

Calculate distribution of shape variable, $R(\alpha_s, Y)$,
using Resummation Calculations:
(Catani, Dokshitzer, Olsson, Trentadue, Turnock, Webber)

Leading and Next-to-Leading logs in
perturbative expansion of $R(\alpha_s, Y)$
summed to all orders of α_s.

$$\text{LL}: \quad L \sum (\alpha_s L)^n \quad , \qquad L \equiv \ln \frac{1}{Y}$$

$$\text{NLL}: \quad \sum (\alpha_s L)^n$$

$\mathcal{O}(\alpha_s^2)$ M.E. and Resummed calculations can be combined.
Must avoid double counting
– some theoretical uncertainty in this

μ dependence should be smaller, since NLL
But NLL calculations not available for so many variables

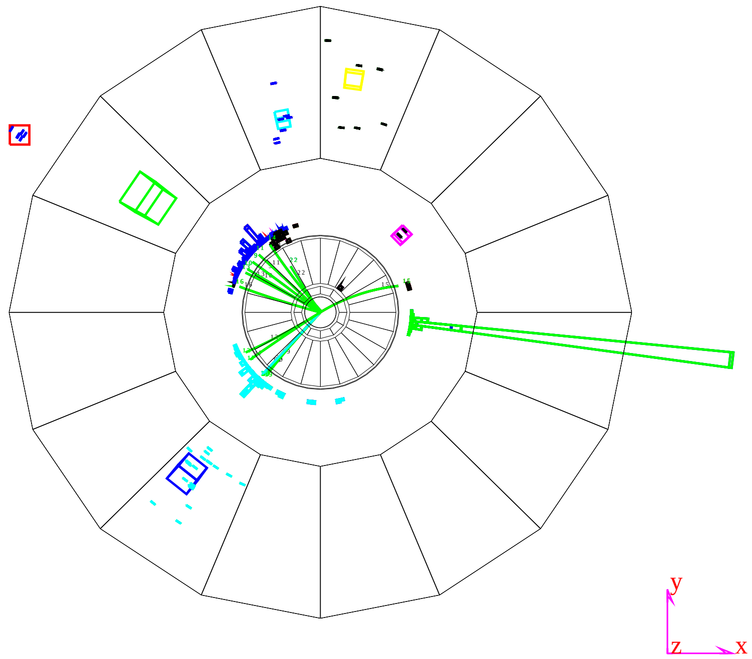
L3 uses this technique from 30 - 183 GeV

L3: Does α_s run ?

Measure α_s at different E_{cms} in one experiment.
Systematic errors largely cancel in test of running

- Z data ($E_{cms} = M_Z$) (142.4 pb^{-1} from 1991–1995)

- Z data with isolated, energetic γ (mostly FSR, ISR)
  - isolated single γ identified by neural net based on shower-shape in BGO
  - Remove the γ from event
  - Boost event to CM of remaining hadronic system $s' = s\left(1 - \frac{2E_\gamma}{\sqrt{s}}\right)$



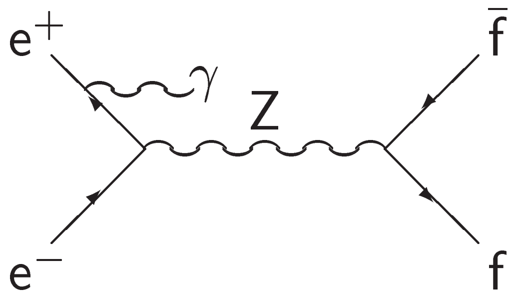
  Efficiency: 27%–48%, Purity:68%–90%

  For QCD, colour field
  equivalent to hadronic event at $E_{cms} = \sqrt{s'} < M_Z$

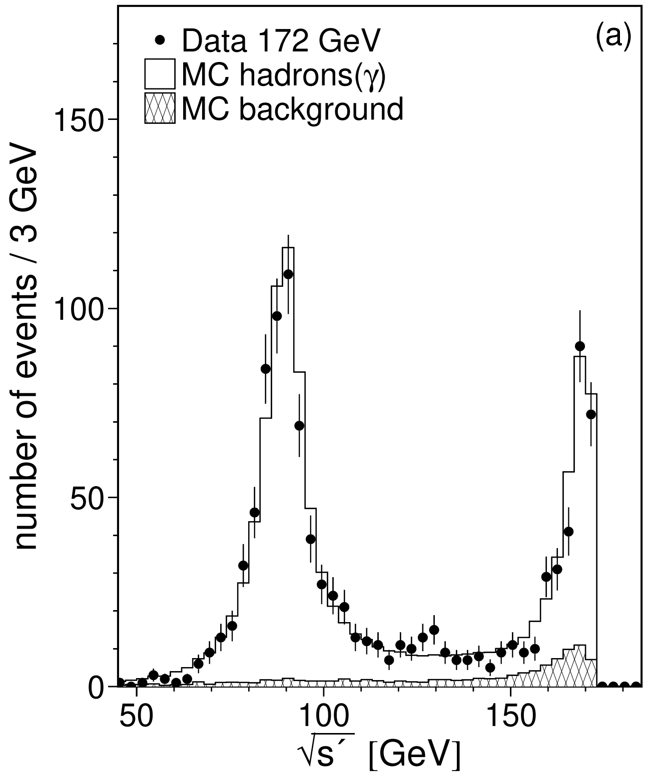
- LEP-2 data at $E_{cms} > M_Z$
  - Reject "Radiative Return" events
  - Reject W^+W^- events

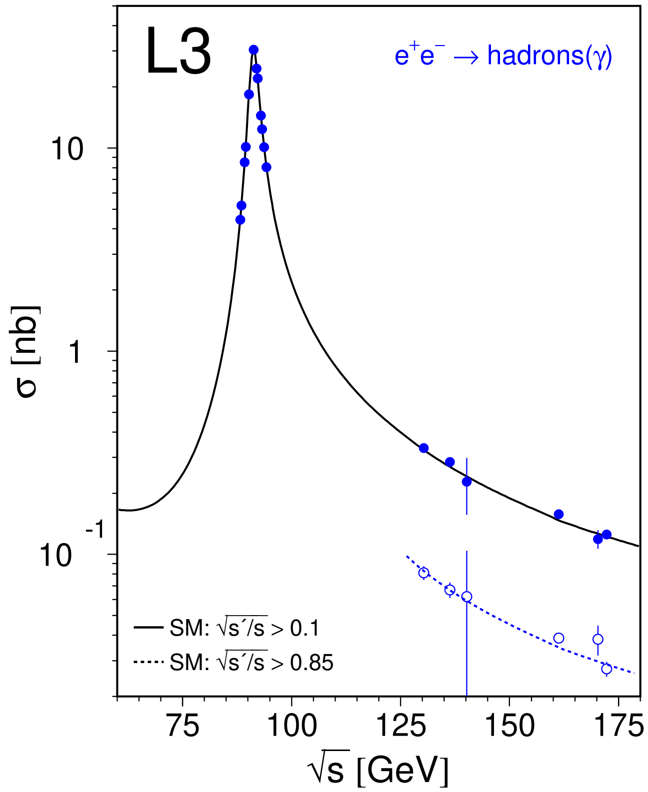
LEP-2: Radiative return to Z

$\mathrm{e^+e^-} \to \gamma/\mathrm{Z} \to \mathrm{f\bar{f}}(\gamma)$, $s = E^2_{\mathrm{cms}} \gg M^2_{\mathrm{Z}}$:
Photon radiated from initial e^- or e^+ (ISR)



Large fraction of $\mathrm{e^+e^-} \to \mathrm{f\bar{f}}(\gamma)$ is radiative return to Z:
Mass $m_{\mathrm{f\bar{f}}} = \sqrt{s'}$ of $\mathrm{f\bar{f}}$ System: $s' = s - 2\,E_\gamma\sqrt{s} \approx M^2_{\mathrm{Z}}$

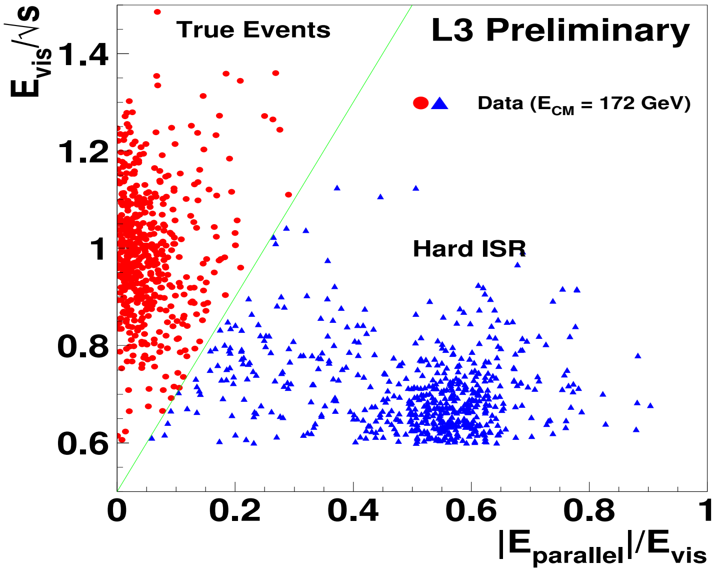




Reject events with high-energy photon
or large $\sum E_{\|}$

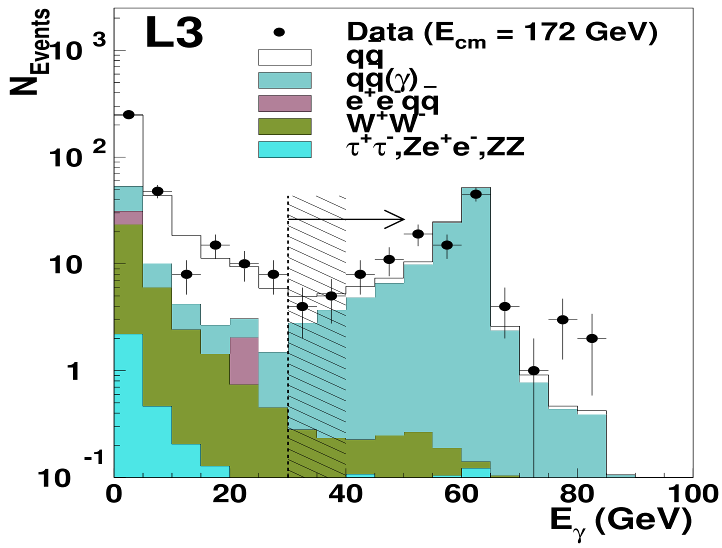
Rejection of radiative return to Z

Reject radiative returns to Z by demanding



low longitudinal energy imbalance :

$$(\mathrm{E_{vis}}/\sqrt{s}) > 2.0\ (|\ \mathrm{E}_{\|}\ |\ /\mathrm{E_{vis}}) + 0.5$$

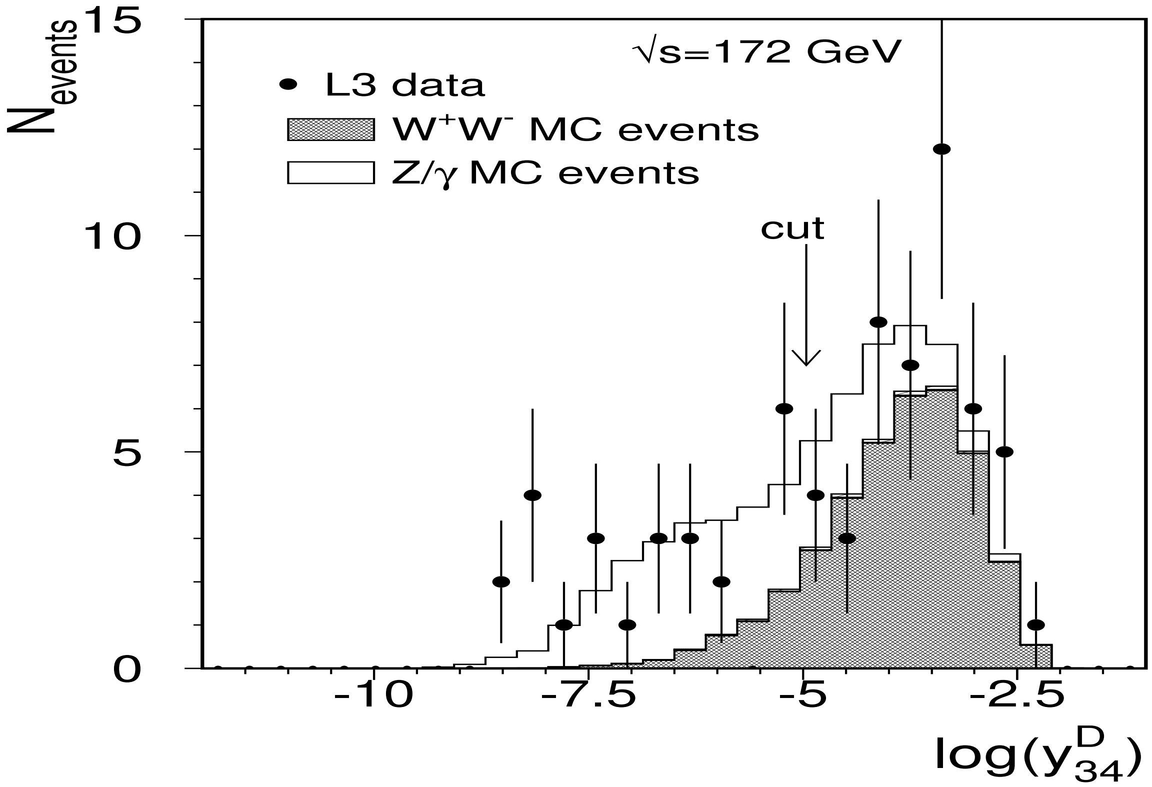


no energetic photon seen in the detector : $\mathrm{E}_\gamma < 30$ GeV

LEP-2: Rejection of W^+W^- events

Identify $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q}$ events by

- Require high multiplicity
- Force into 4-jet configuration (Durham algorithm)
- Select good 4-jet events by cuts on E of most and least energetic jets
- Cut on jet resolution parameter y_{34}



Selection efficiency of $e^+e^- \to W^+W^- \to q\bar{q}q\bar{q} \approx 90\%$

Reject these events.

α_s from shape variables, III

L3 uses shape variables:

- 1 − Thrust:

$$1 - T = 1 - \max \frac{\sum |\vec{p} \cdot \vec{n}_\mathsf{T}|}{\sum |\vec{p}|}$$

- Scaled Heavy Jet Mass: hemispheres defined by $\vec{n}_\mathsf{T}$

$$\begin{aligned} M_\pm^2 &= \left(\sum_{i \in \mathsf{hemi}\pm} p_i \right)^2 \\ M_\mathsf{H} &= \max\left(M_+, M_-\right) \\ \rho &= M_\mathsf{H} / E_\mathsf{cms} \end{aligned}$$

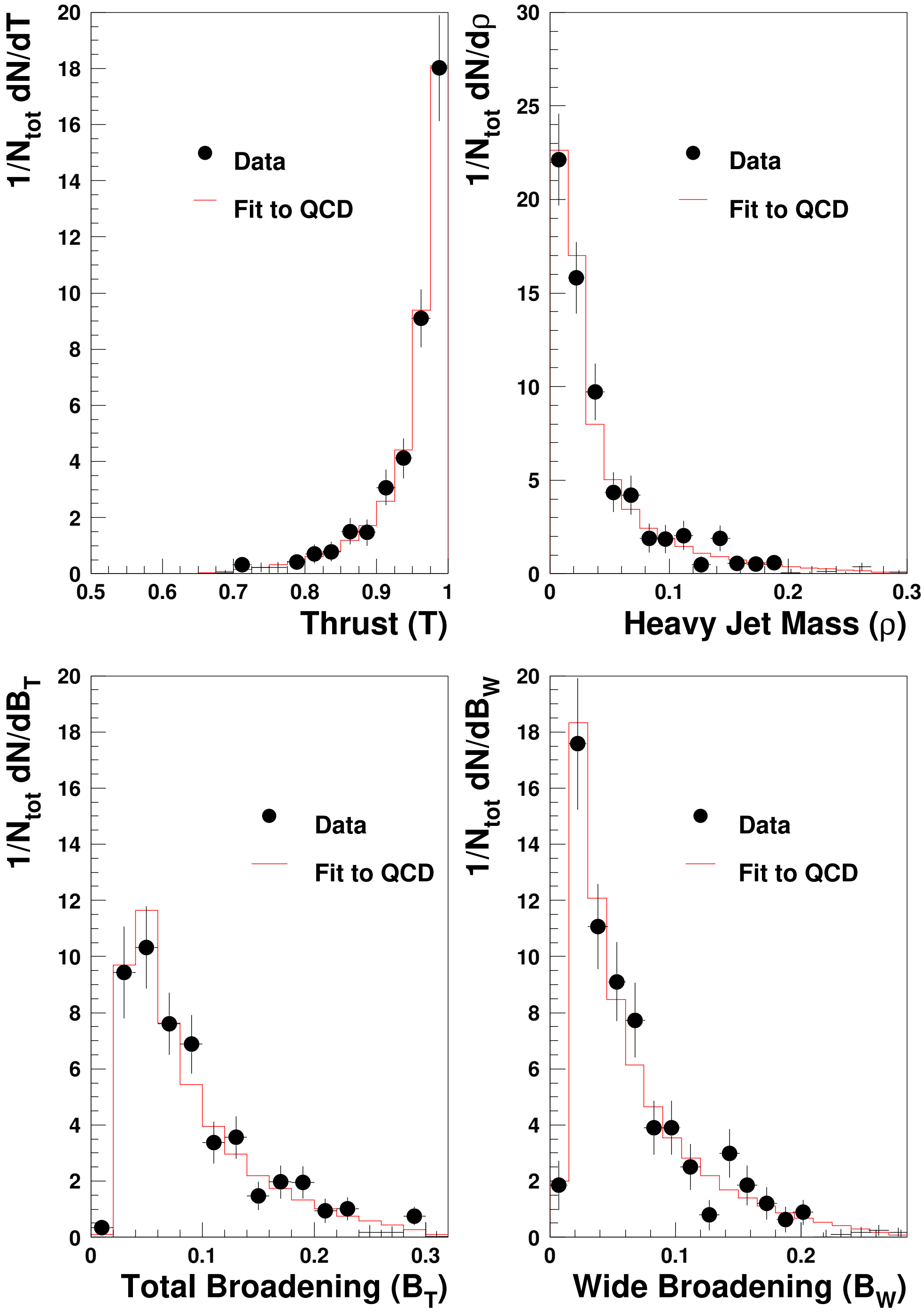
- Jet Broadening Variables:

$$B_\pm = \frac{\sum_{i \in \mathsf{hemi}\pm} |\vec{p}_i \times \vec{n}_\mathsf{T}|}{2 \sum_i |\vec{p}_i|}$$

Total Jet Broadening: $B_\mathsf{T} = B_+ + B_-$
Wide Jet Broadening: $B_\mathsf{W} = \max\left(B_+, B_-\right)$

α_s from shape variables, III

L3 data at 172 GeV



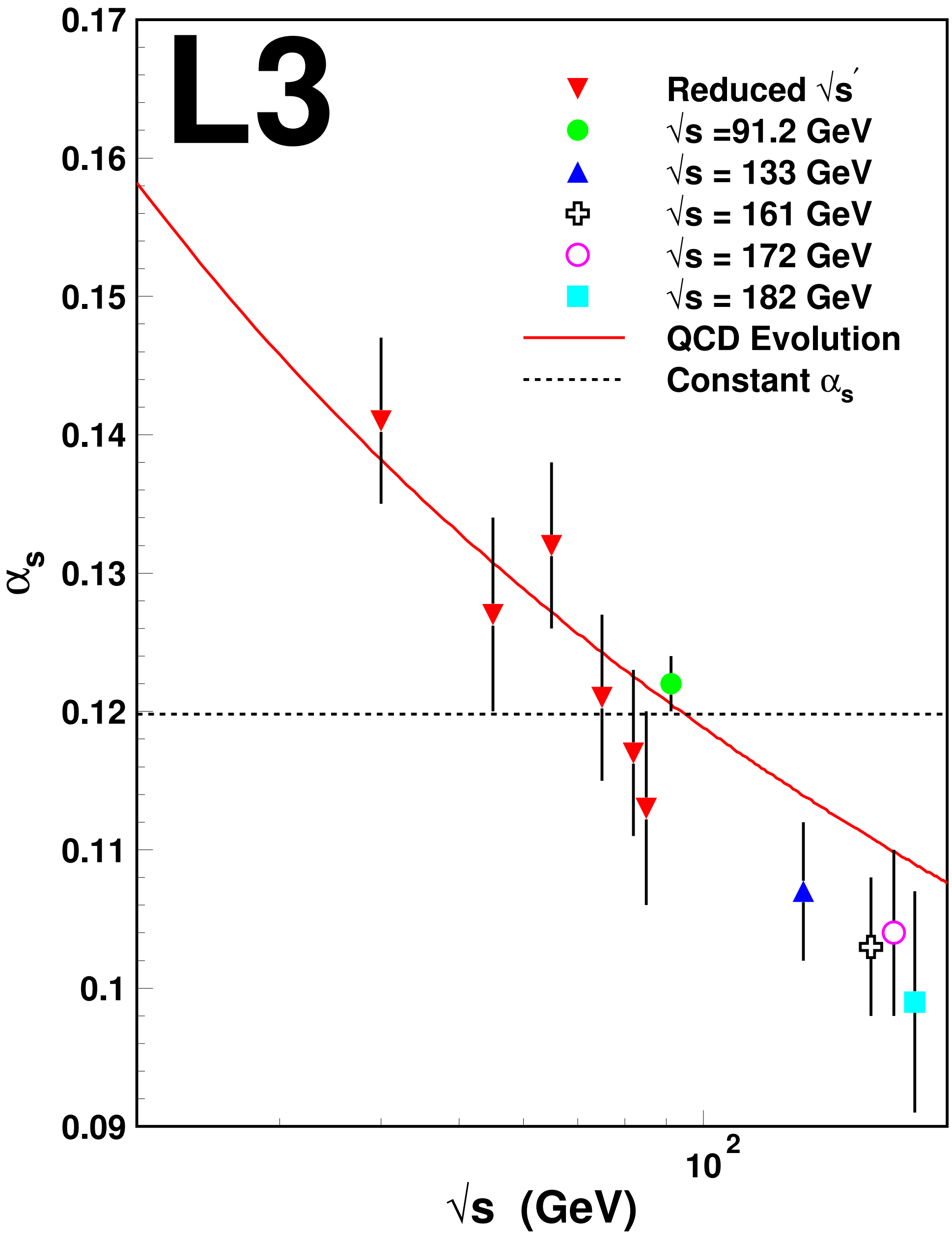
Reasonable fits to anal. dists. with one free parameter, α_s

Mean of 4 α_s values:

$$\alpha_s(172\,\text{GeV}) = 0.104 \pm 0.006 \pm 0.005$$

$$\alpha_s(M_Z) = 0.114 \pm 0.007$$

L3: Does α_s run ?



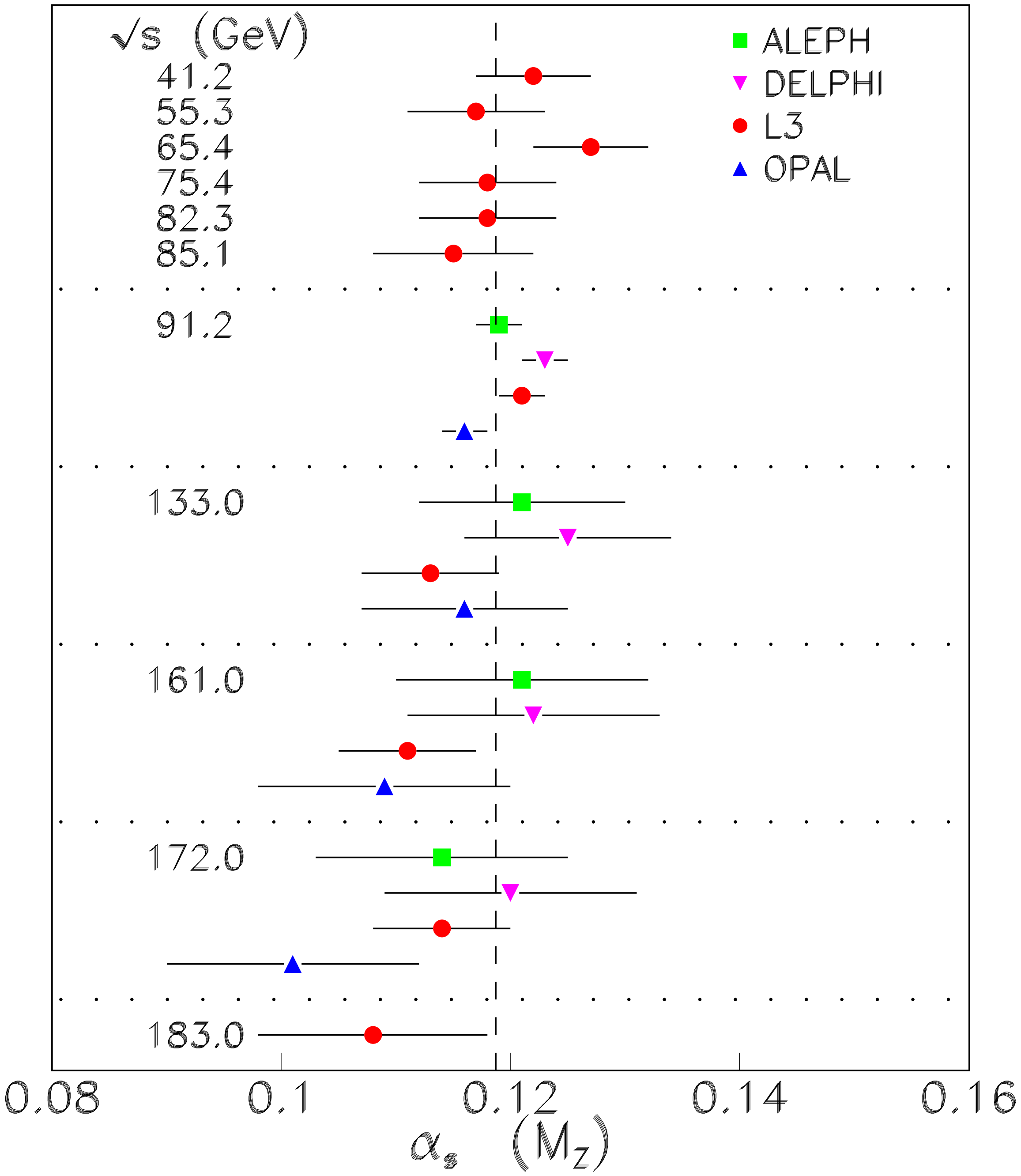
Fit to the 3-loop QCD evolution equation, using experimental errors only, gives

$$\alpha_s(M_Z) = 0.1205 \pm 0.0016 \text{ (expt) } \pm 0.0066 \text{ (theory)}$$

$$\chi^2 = 12.6 \text{ for } 10 \text{ DoF}, \ \text{CL} = 0.25$$

α_s from shape variables, III

Other LEP expts. have also analyzed shape variables using $\mathcal{O}(\alpha_s^2)$ M.E. + Resummed LL, NLL.



Avg.: $\alpha_s(M_Z) = 0.1188 \pm 0.0009$ (expt)
± 0.0066 (theory)

Gluon Splitting

DELPHI (CERN-PPE/97-039): in LEP-I 4-jet events, measure

$$g_{\mathrm{b\bar{b}}} = \frac{N(\mathrm{Z} \to \mathrm{q\bar{q}g},\ \mathrm{g} \to \mathrm{b\bar{b}})}{N(\mathrm{Z} \to \mathrm{hadrons})}$$

b is heavy $\Longrightarrow$ $g_{\mathrm{b\bar{b}}}$ small

Important: uncertainty on $g_{\mathrm{b\bar{b}}}$ gives
large syst. error on $R_{\mathrm{b}} = \frac{\Gamma_{\mathrm{b\bar{b}}}}{\Gamma_{\mathrm{had}}}$

Assume 2 jets with smallest angle between them are $\mathrm{g} \to \mathrm{q\bar{q}}$ (MC: 54% eff.)

Identify jets as b using lifetime (vertex) tag

Difficult analysis with lots of MC to get efficiencies, purities.

Use OPAL result $g_{\mathrm{c\bar{c}}} = 2.27\%$ to estimate contamination from $\mathrm{c\bar{c}}$

ALEPH (EPS97 conf.) has done a similar analysis

Results:

DELPHI	$g_{\mathrm{b\bar{b}}} = 0.21 \pm 0.11 \pm 0.09\%$
ALEPH	$g_{\mathrm{b\bar{b}}} = 0.26 \pm 0.04 \pm 0.09\%$
$\mathcal{O}(\alpha_s^2)$+resummed LL, NLL	$g_{\mathrm{b\bar{b}}} = 0.18\%$
JETSET PS	$g_{\mathrm{b\bar{b}}} = 0.16\%$

Monte Carlo models

MC models are important:

- to correct data – e.g., corrections for hadronization effects, calculations of efficiencies, acceptances

- to test theoretical ideas – e.g., LLA, NLLA calculations are generally for massless quarks (or $s \to \infty$). MCs incorporate masses, E conservation, etc. (although perhaps not correctly)

- to gain insight by using (toy) models – e.g., in non-perturbative regions.

MCs for $e^+e^- \to$ hadrons:

PYTHIA JETSET	$\mathcal{O}(\alpha_s^2)$ M.E.	string frag. (JETSET)
PYTHIA JETSET	∼MLLA shower	string frag. (JETSET)
ARIADNE	∼MLLA shower	string frag. (JETSET)
HERWIG	∼MLLA shower	cluster frag.
COJETS	∼LLA shower	independent frag.

Using MC tuned at Z(LEP-I), compare shape variable Y

1. Y dist. to MC at high energy
2. evolution of $< Y >$ with $\sqrt{s}$

many variables – will not show all

Shape Variables – defs.

Thrust T: This observable is defined by the expression [10]

$$T = \max_{\vec{n}} \left(\frac{\sum_i |p_i \cdot \vec{n}|}{\sum_i |p_i|} \right) \quad . \tag{1}$$

The thrust axis $\vec{n}_T$ is the direction $\vec{n}$ which maximises the expression in parentheses. A plane through the origin and perpendicular to $\vec{n}_T$ divides the event into two hemispheres H_1 and H_2.

Thrust major T_{maj}: The maximisation in equation 1 is performed with the condition that $\vec{n}$ must lie in the plane perpendicular to $\vec{n}_T$. The resulting vector is called $\vec{n}_{T_{\mathrm{maj}}}$.

Thrust minor T_{min}: The expression in parentheses is evaluated for the vector $\vec{n}_{T_{\mathrm{min}}}$ which is perpendicular both to $\vec{n}_T$ and to $\vec{n}_{T_{\mathrm{maj}}}$.

Oblateness O: This observable is defined by $O = T_{\mathrm{maj}} - T_{\mathrm{min}}$ [11].

Sphericity S and Aplanarity A: These observables are based on the momentum tensor

$$S^{\alpha\beta} = \frac{\sum_i p_i^\alpha p_i^\beta}{\sum_i p_i^2} \quad , \quad \alpha, \beta = 1, 2, 3 \quad .$$

The three eigenvalues Q_j of $S^{\alpha\beta}$ are ordered such that $Q_1 < Q_2 < Q_3$. These then define S [12] and A [13] by

$$S = \frac{3}{2}(Q_1 + Q_2) \quad \text{and} \quad A = \frac{3}{2} Q_1 \quad .$$

C-parameter C: The momentum tensor $S^{\alpha\beta}$ is linearised to become

$$\Theta^{\alpha\beta} = \frac{\sum_i (p_i^\alpha p_i^\beta)/|p_i|}{\sum_i |p_i|} \quad , \quad \alpha, \beta = 1, 2, 3 \quad .$$

The three eigenvalues λ_j of this tensor define C [14] with

$$C = 3(\lambda_1\lambda_2 + \lambda_2\lambda_3 + \lambda_3\lambda_1) \quad .$$

Heavy Jet Mass M_H: The hemisphere invariant masses are calculated using the particles in the two hemispheres H_1 and H_2. We define M_H [15] as the heavier mass, divided by $\sqrt{s}$.

Jet Broadening variables B_T and B_W: These are defined by computing the quantity

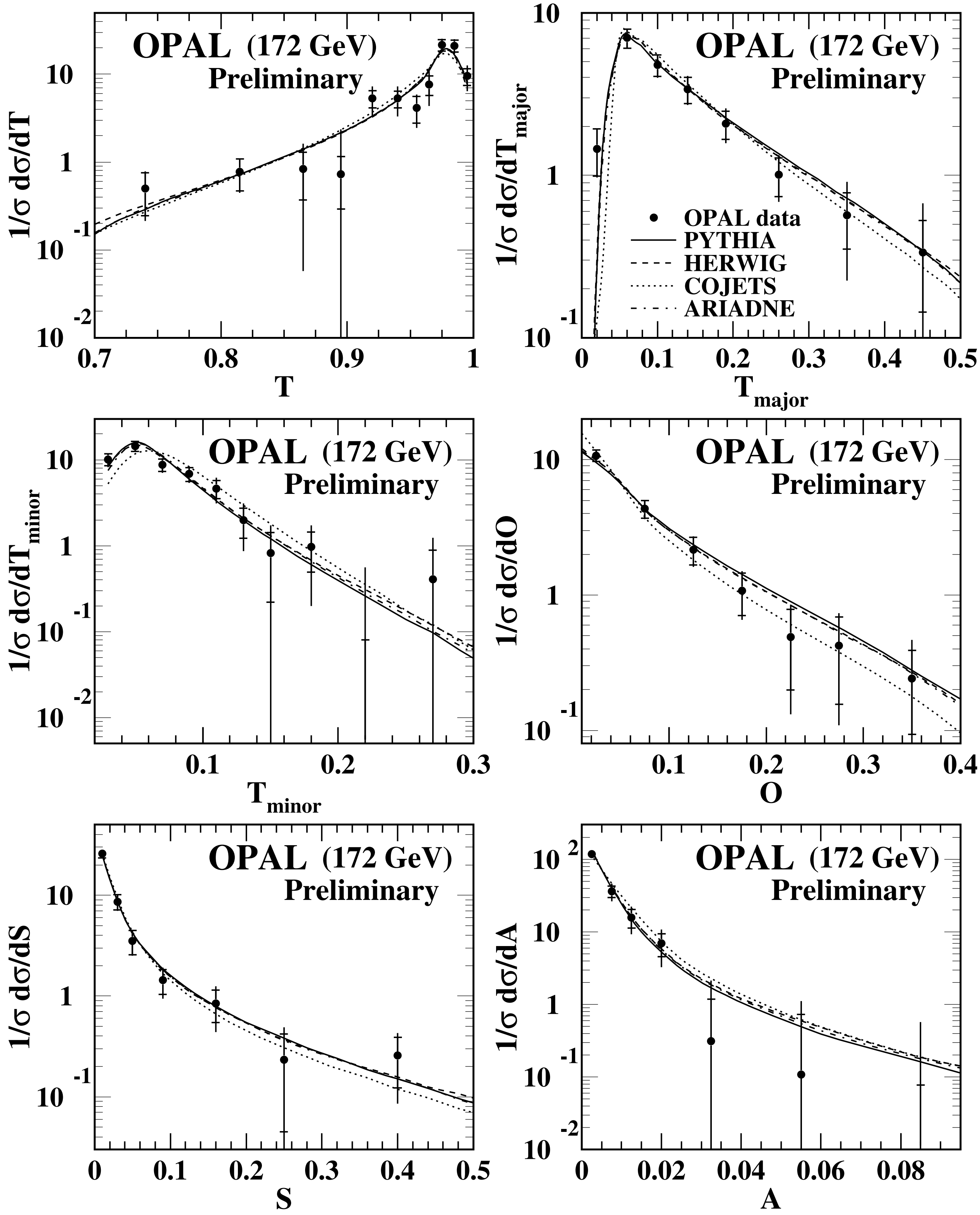
$$B_k = \left(\frac{\sum_{i \in H_k} |p_i \times \vec{n}_T|}{2 \sum_i |p_i|} \right)$$

for each of the two event hemispheres, H_k, defined above. The two observables [16] are defined by

$$B_T = B_1 + B_2 \quad \text{and} \quad B_W = \max(B_1, B_2)$$

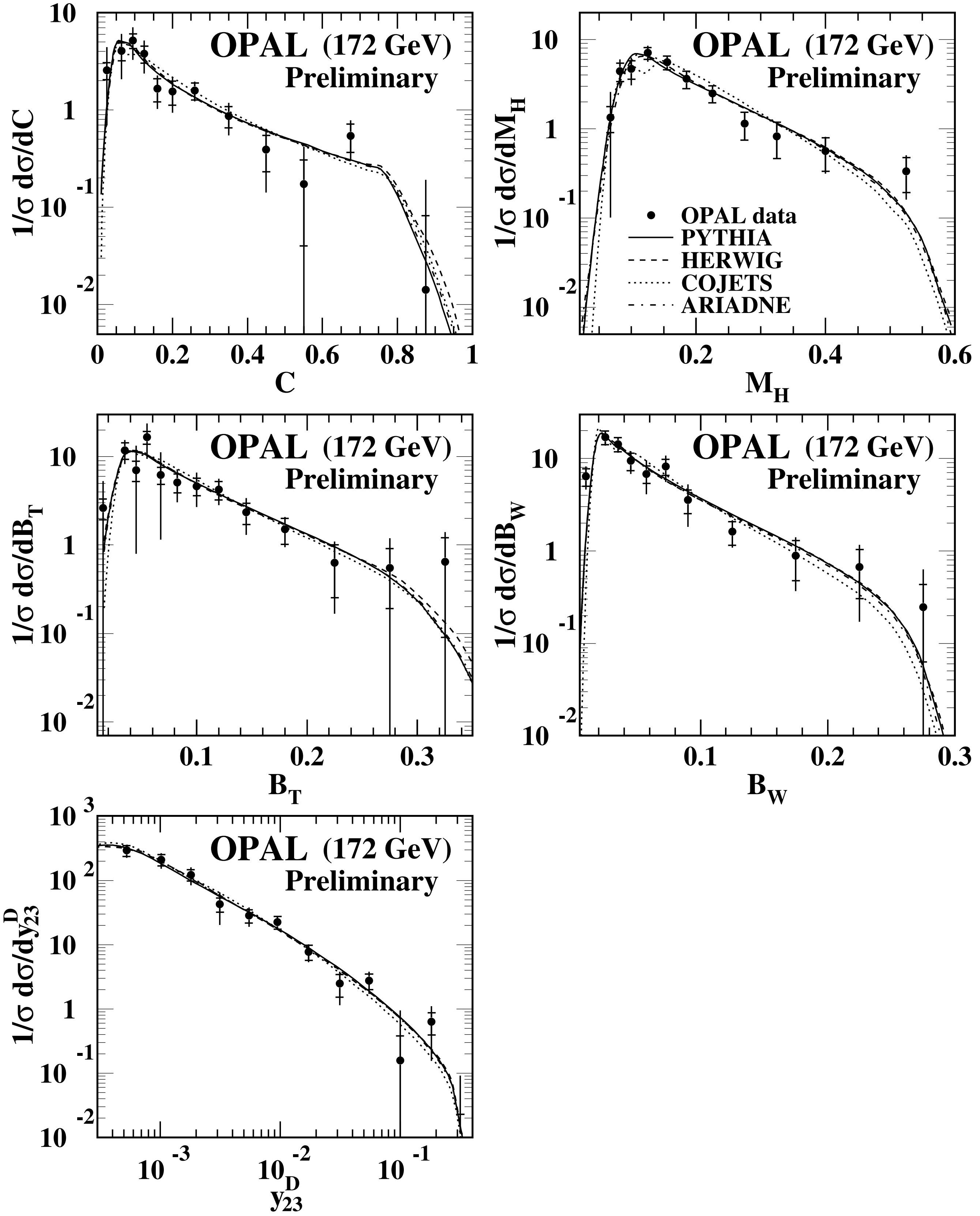
where B_T is the total and B_W is the wide jet broadening.

Shape Variables



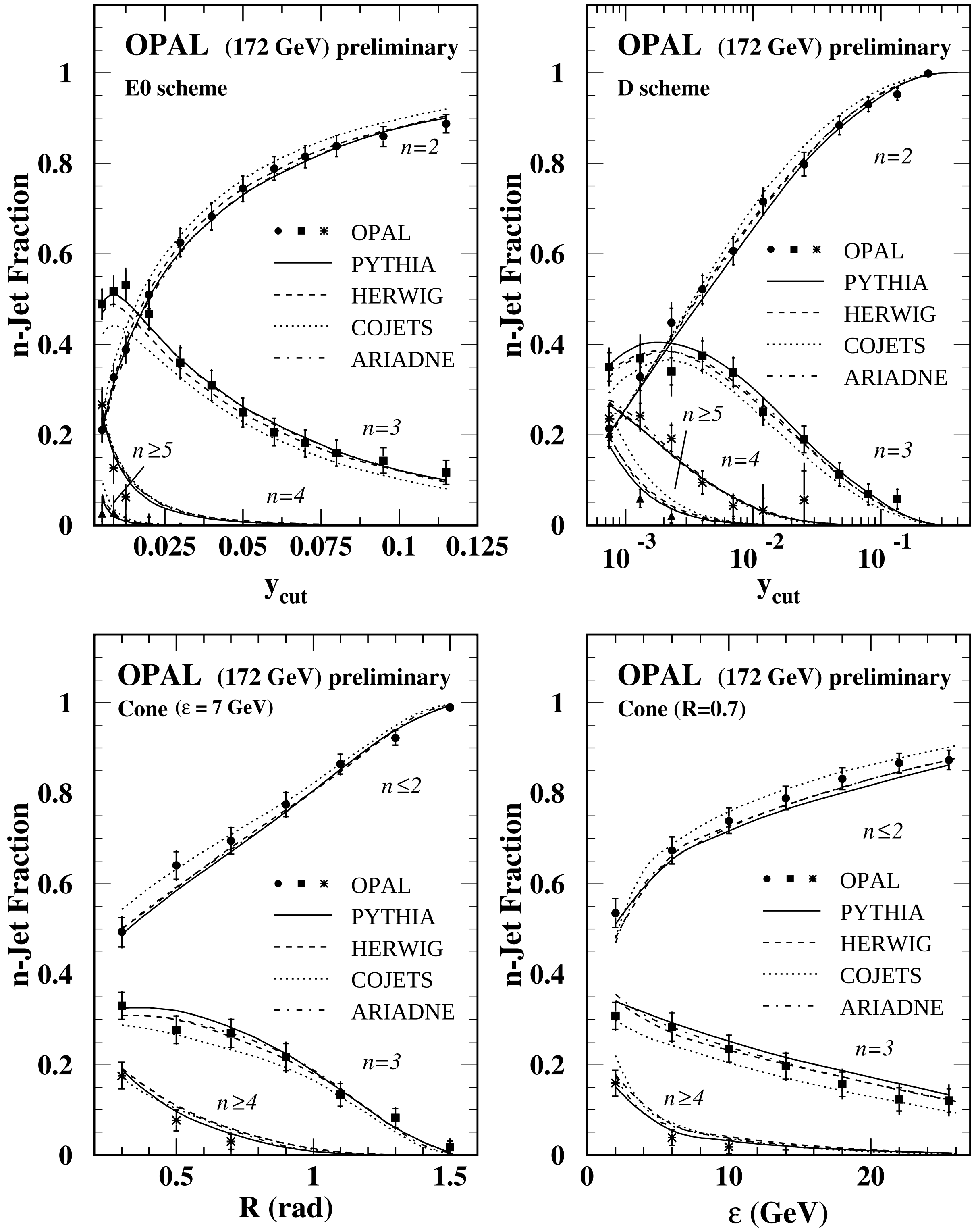
Good agreement

Shape Variables



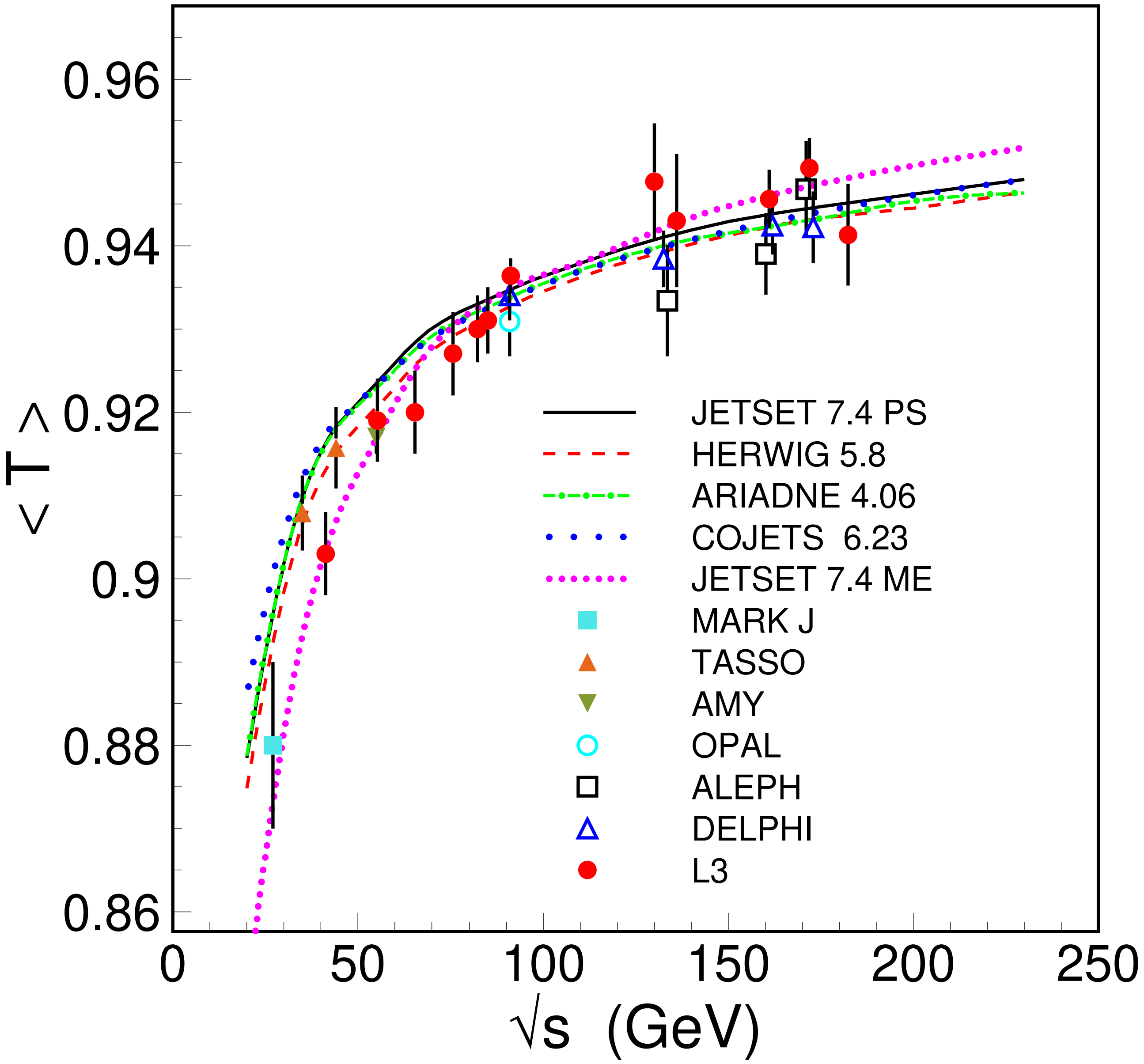
Good agreement

Jet Fractions



Good agreement

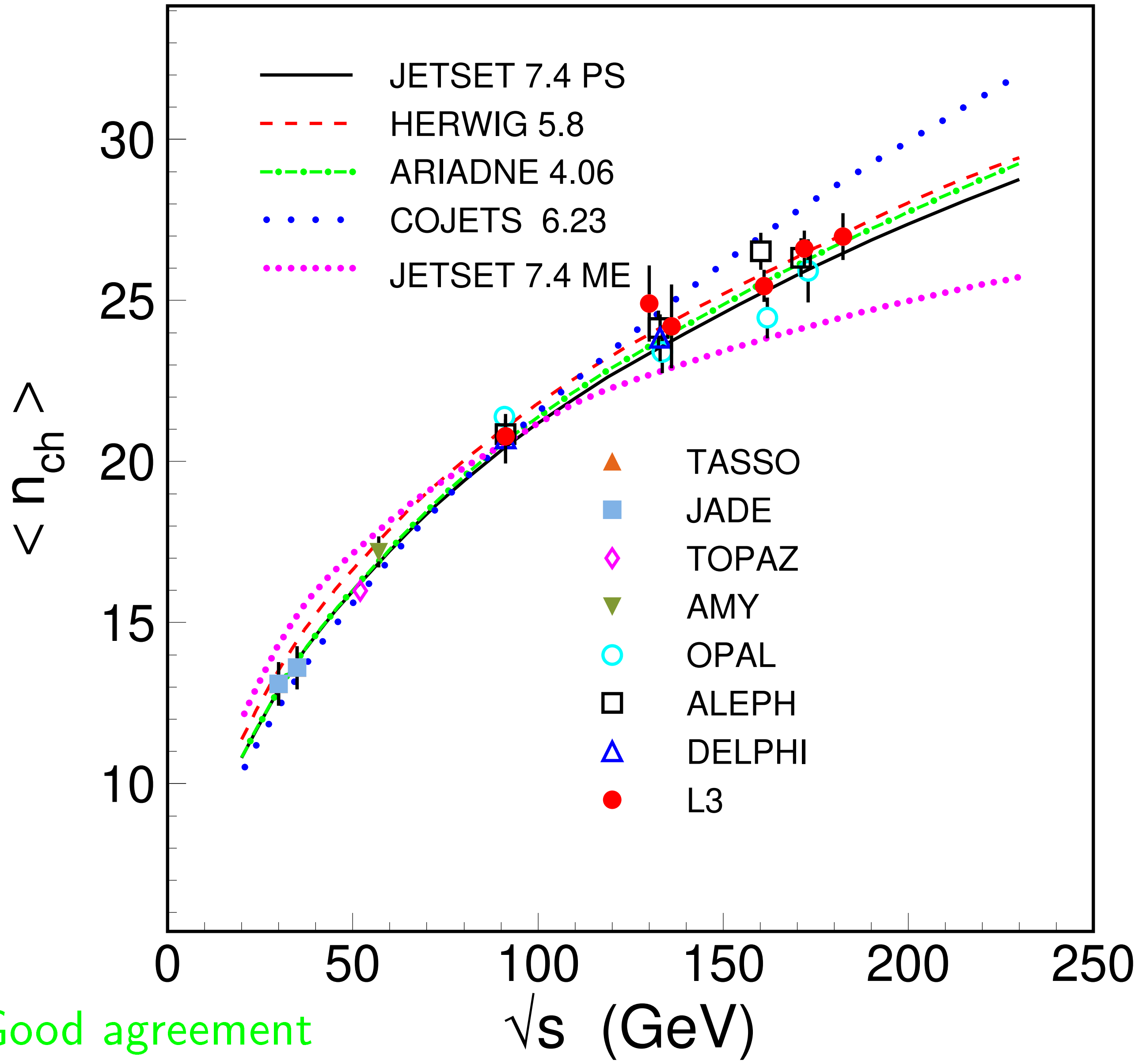
Shape Variables – Evolution



Good agreement

Other shape variables – even better

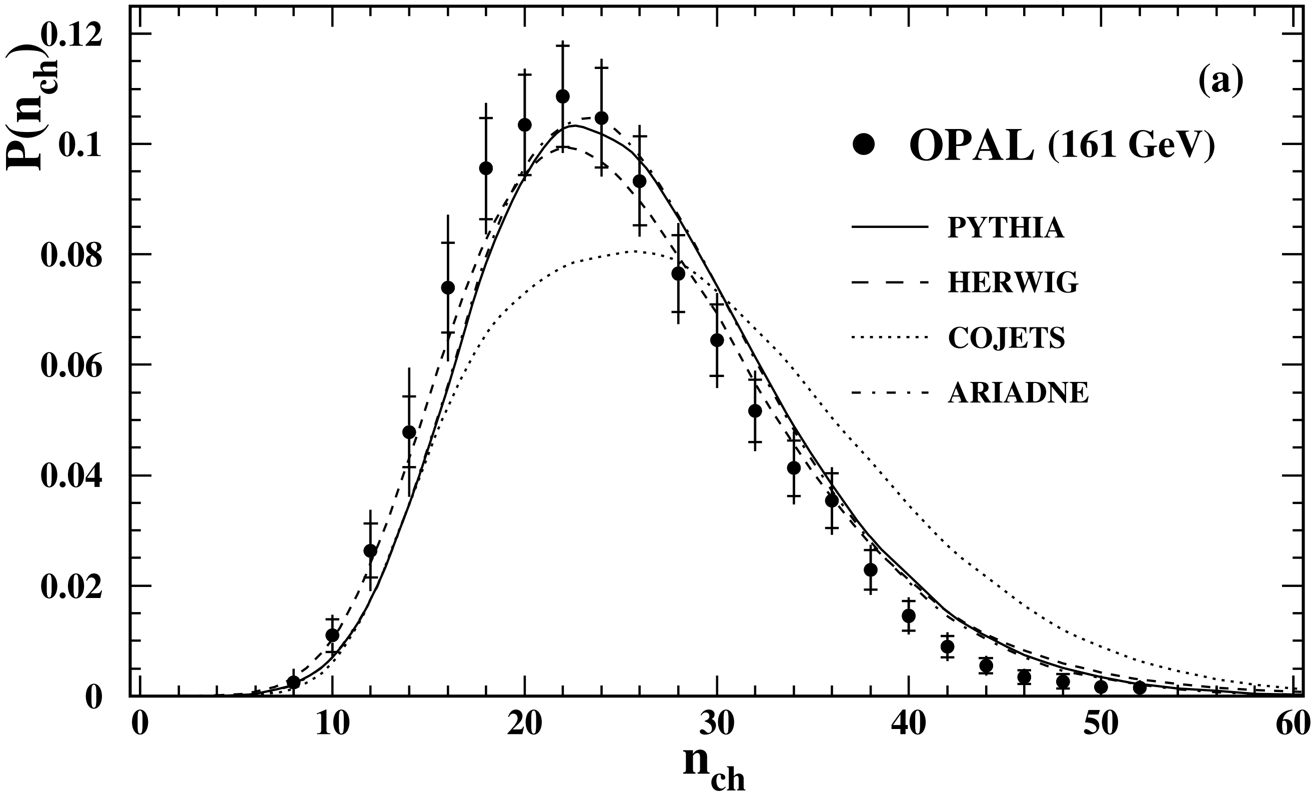
Charged-Particle Multiplicity



Good agreement
except for COJETS, JETSET M.E.

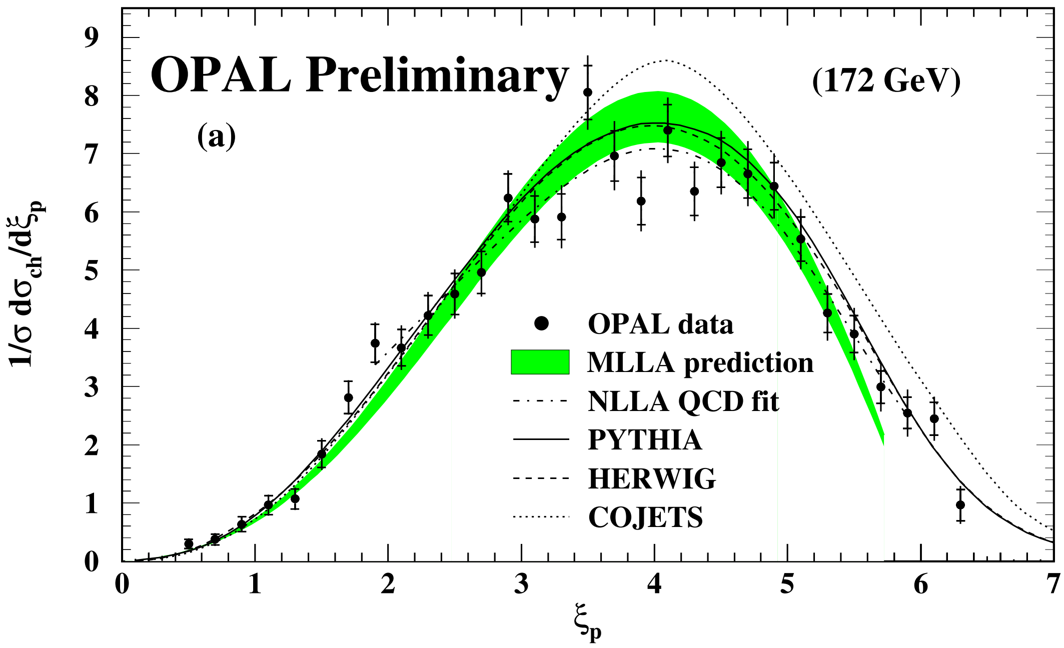
- COJETS disagrees because it has only LLA shower.
  Need MLLA shower, which incorporates
  soft gluon destructive interference.
- JETSET M.E. disagreement not surprising
  String fragmentation must simulate soft gluons.
  Number of soft gluons depends on $E_{\rm cm}$,
  so must be retuned at each $E_{\rm cm}$.

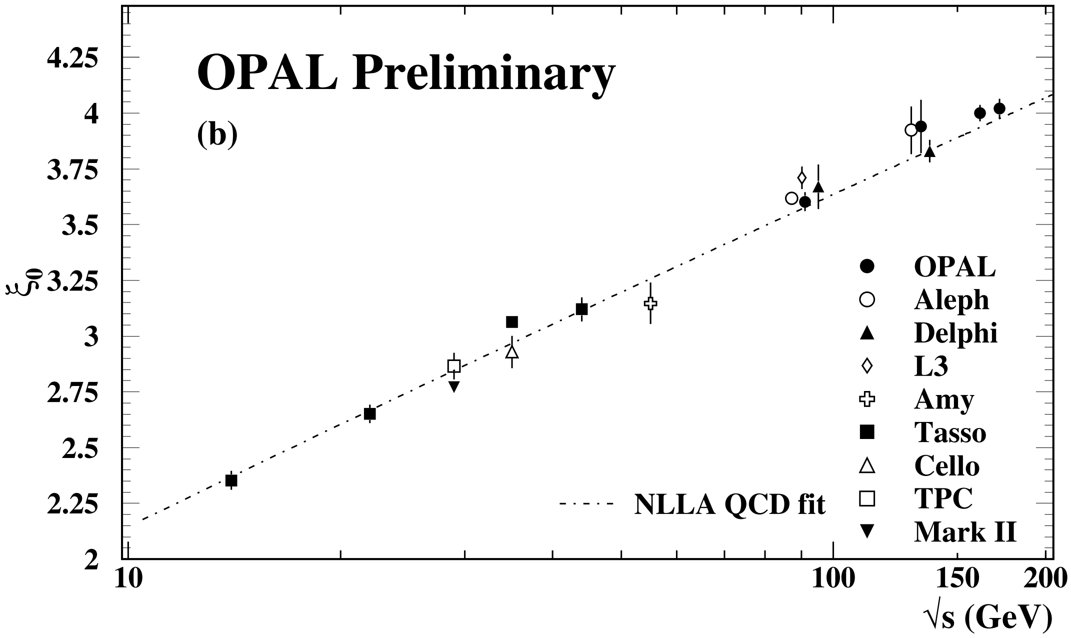
Charged-Particle Multiplicity



Good agreement
except for COJETS

Single-Particle Spectra





(a) COJETS (LLA) far off
PYTHIA, HERWIG, MLLA a bit too high at max.
MLLA drops a bit too fast at high $\xi_p = -\ln x_p = \frac{2\,p}{E_{\text{cms}}}$
NLLA Good agreement
So, MLLA begins not to be sufficient.

(b) NLLA describes well ξ_0 evolution with $\sqrt{s}$.
($\xi_0 = \xi$ at which $\mathrm{d}\sigma/\mathrm{d}\xi_p$ is max.)

Shape-variable Evolution

Conclusions from shape-variable evolution:

- Coherent parton shower ($\sim$MLLA) confirmed by data
- MLLA/NLLA analytic calculations also confirmed for single-particle spectra

  But note:

  – These calculations are for gluon radiation.
  – Application to particles requires assumption of Local Parton Hadron Duality (LPHD):

  $$f_{\text{particle}}(Y) \propto f_{\text{parton}}(Y)$$

  an appealing, if simple, idea – more later

Quark-Gluon jet differences

Larger gluon colour charge $\left(\mathrm{g/q}: \frac{C_\mathrm{A}}{C_\mathrm{F}} = \frac{9}{4}\right)$

$\Longrightarrow$ gluons radiate more soft gluons than quarks do

LL: $\frac{<N_\mathrm{g}>}{<N_\mathrm{q}>} = \frac{C_\mathrm{A}}{C_\mathrm{F}} = \frac{9}{4}$

$\Longrightarrow$ more particles in a gluon jet than in a quark jet

LL+LPHD: $\frac{<N_\mathrm{g}>}{<N_\mathrm{q}>} = \frac{C_\mathrm{A}}{C_\mathrm{F}} = \frac{9}{4}$

$\Longrightarrow$ gluons in a gluon jet are softer than in a quark jet

$\Longrightarrow$ particles in a gluon jet are softer than in a quark jet

But comparison of q and g jets is difficult

Quark-Gluon jet differences

Comparison is difficult. To use $e^+e^- \to q\bar{q}g$ events,

- want unbiased method of identifying q
  use b-tagging – usually decay vertex

- want $E_q \approx E_g$ select
  - "Mercedes" topology:
    $E_1 \approx E_2 \approx E_3$ (jets 1 and 2 with b-tags)
  - "Y" topology:
    $E_1 > E_2 \approx E_3$ (jet-2 with b-tag, assume jet 1 is q)

- But soft gluon interference in PS depends on angles between q, g jets

- Analytical calculations of properties of q, g jets are for
  - $q\bar{q}$ events and for gg events.
    q or g jet is then 1 hemisphere of the event.
  - But, we do NOT have gg events.

2 new approaches to using $e^+e^- \to q\bar{q}g$ events:

- DELPHI: Different scales depending on jet angles for q,g.

- OPAL: Events which approximate gg events.

Charged Particle Multiplicity — q/g

DELPHI (EPS97 conf.) adopts a
"hardness scale" for quarks: (Dokshitzer, Diakonov, Trojan)

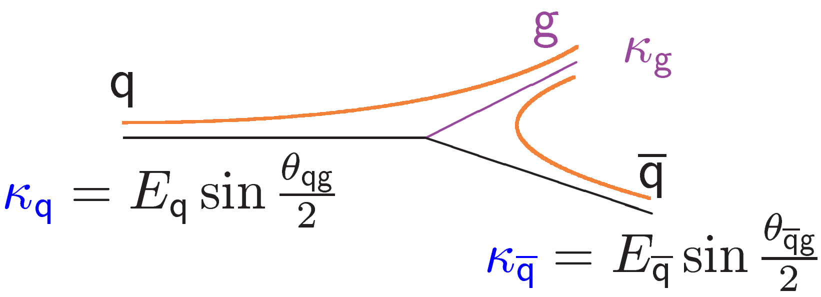
$$\kappa_{\mathrm{DDT}} = E_{\mathrm{jet}} \sin\frac{\theta}{2}$$

$\theta =$ angle between jet and closest other jet

$\kappa_{\mathrm{DDT}} \approx p_{\mathrm{t}}$ of process producing the parton

For 2-jet event, $\theta = 180^\circ \Longrightarrow \kappa = \kappa_{\mathrm{DDT}} \equiv E_{\mathrm{jet}} = E_{\mathrm{cms}}/2$

For gluons:



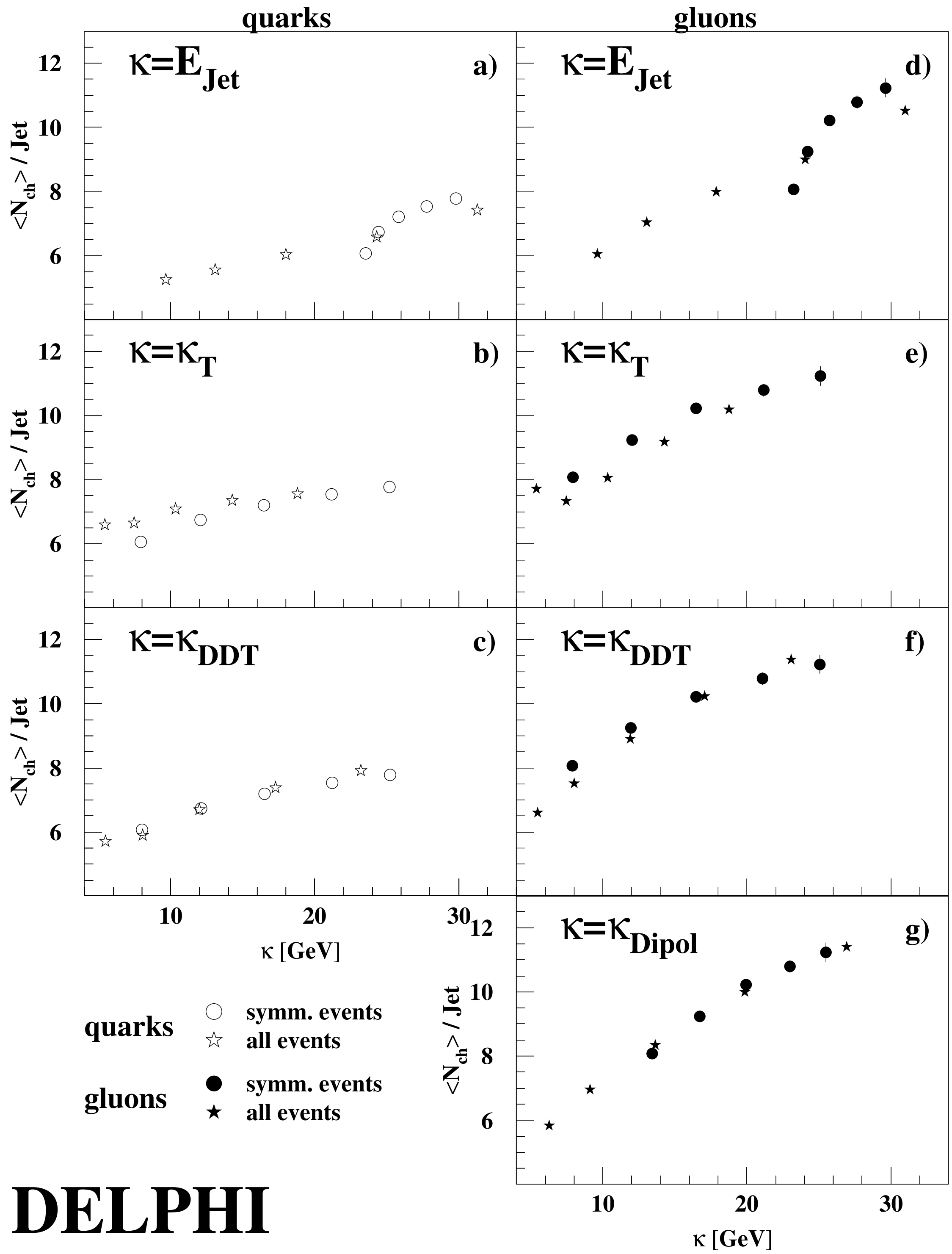
From dipole model colour fields, angular dependence of κ_{g} should be some kind of average of κ_{q}, $\kappa_{\bar{\mathrm{q}}}$

– scale usually enters calculations via logs

– $\therefore$ use "average of logs" $\Rightarrow$ harmonic mean:

$$\kappa_{\mathrm{g}} = \kappa_{\mathrm{dipole}} \equiv E_{\mathrm{g}} \sqrt{\sin\frac{\theta_{\mathrm{qg}}}{2} \sin\frac{\theta_{\bar{\mathrm{q}}\mathrm{g}}}{2}}$$

Charged Particle Multiplicity — q/g



Topological dependence is taken into account by κ_{DDT} for quarks and κ_{dipole} for gluons

Charged Particle Multiplicity — q/g

Fit to
NNLO analytic prediction
+ LPHD assumption
+ constant for non-perturbative effects
using κ_{DDT} for quarks and κ_{dipole} for gluons

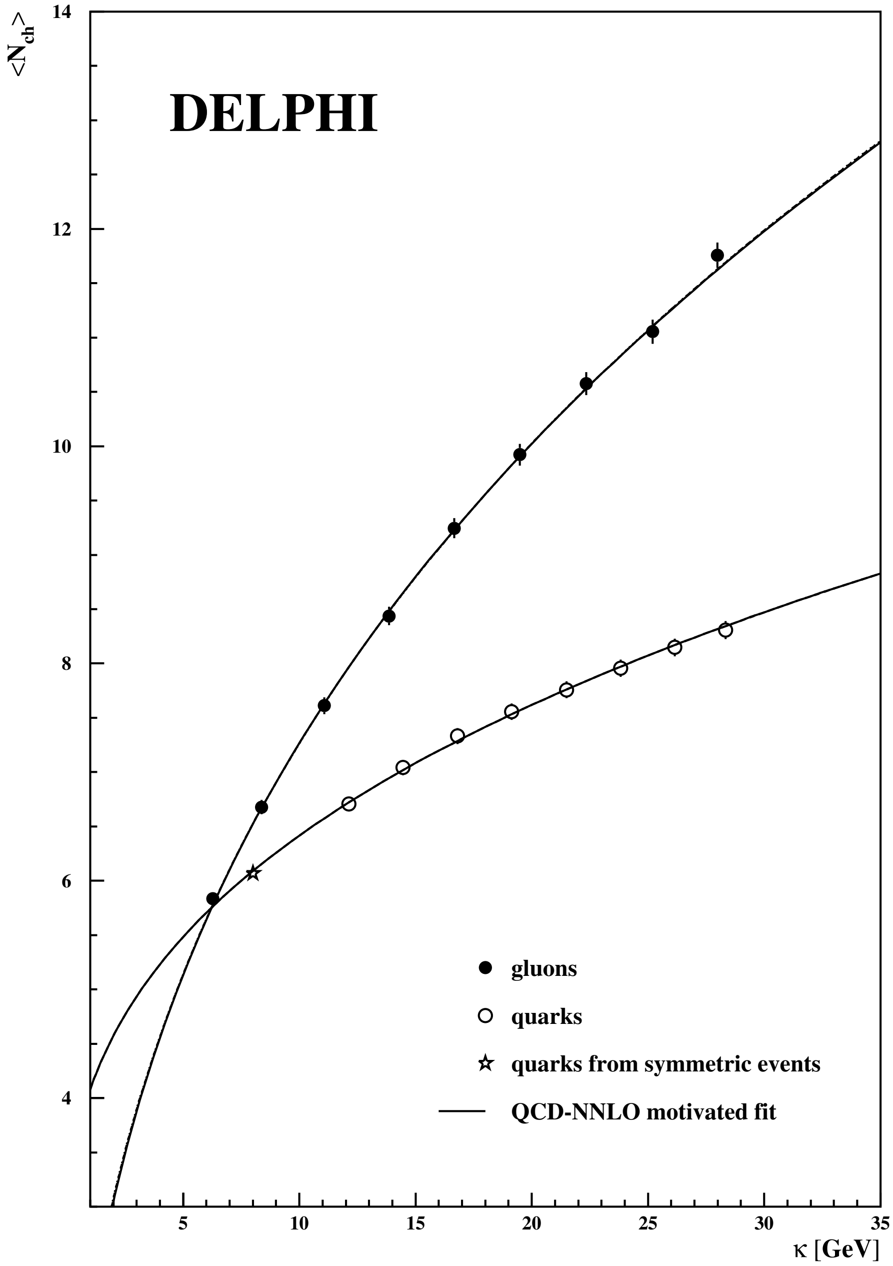
$$
\begin{aligned}
< N_{\mathsf{q}} > &= N + N_{\mathsf{q}}^{\mathsf{non-pert}} \\
< N_{\mathsf{g}} > &= N r(\kappa) + N_{\mathsf{g}}^{\mathsf{non-pert}} \\
\text{where } N &= K \alpha_{\mathsf{s}}^{b} \exp \frac{c}{\sqrt{\alpha_{\mathsf{s}}}} [1 + \mathcal{O}(\sqrt{\alpha_{\mathsf{s}}})] \\
r(\kappa) &= \frac{C_{\mathsf{A}}}{C_{\mathsf{F}}} \left(1 - r_1 \gamma_0 - r_2 \gamma_0^2\right)
\end{aligned}
$$

r_1, r_2 are the NL, NNL terms
b, c, r_1, r_2 are known functions of C_{A}, C_{F}, n_{f}, α_{s}.

Thus 3 fit params.: K, $N_{\mathsf{q}}^{\mathsf{non-pert}}$, $N_{\mathsf{g}}^{\mathsf{non-pert}}$

Also take $\frac{C_{\mathsf{A}}}{C_{\mathsf{F}}}$ as fit param. to measure it

Charged Particle Multiplicity — q/g



Good fit

Success for NLLA with

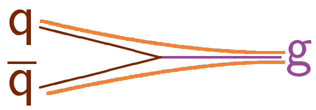
κ_{DDT}, κ_{dipole}

$+$ LPHD

	$\frac{C_A}{C_F}$	
Fit:	$2.34 \pm 0.04 \pm 0.11$	
SU(3)	2.25	agrees well
ALEPH 4-jet	$2.20 \pm 0.09 \pm 0.13$	(CERN-PPE/97-002)
DELPHI 4-jet	$2.51 \pm 0.25 \pm 0.16$	(EPS97 236)

Charged Particle Multiplicity — q/g

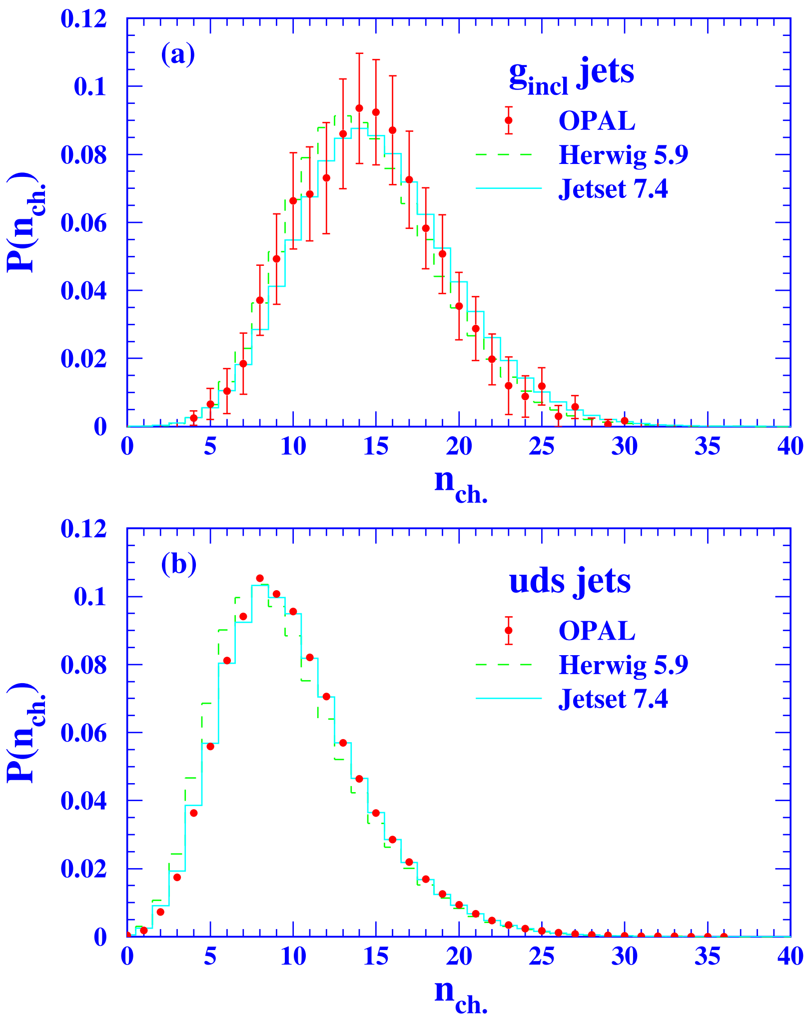
OPAL selects g jet in $q\bar{q}g$ events where $q\bar{q}$ close together (10-50°) and both tagged as b (294 events).



Then color field ≈ that in a "gg" event.

They then compare the g with a q from a 2-jet ($q\bar{q}$) event with a light-quark tag (since QCD calculations are for massless quarks)

$\langle E_g \rangle = 41.2$ GeV (from MC) $\approx E_q = E_{beam} = 45.6$ GeV



Good agreement with JETSET, HERWIG

Charged Particle Multiplicity — q/g

Correcting q mult. to 41.2 GeV using QCD $\langle n_{\mathrm{ch}} \rangle$ evolution,

$$r \equiv \frac{\langle n_{\mathrm{ch}} \rangle_{\mathrm{g}}}{\langle n_{\mathrm{ch}} \rangle_{\mathrm{uds}}} = 1.494 \pm 0.025 \pm 0.051$$

Factorial and cumulant moments of the mult. dists. and their ratio:

$$F_q = \frac{\sum_{n=q}^{\infty} \frac{n!}{(n-q)!} P_n}{\left(\sum_{n=1}^{\infty} n\, P_n\right)^q}$$

$$K_q = F_q - \sum_{m=1}^{q-1} \binom{q-1}{m} K_{q-m} F_m$$

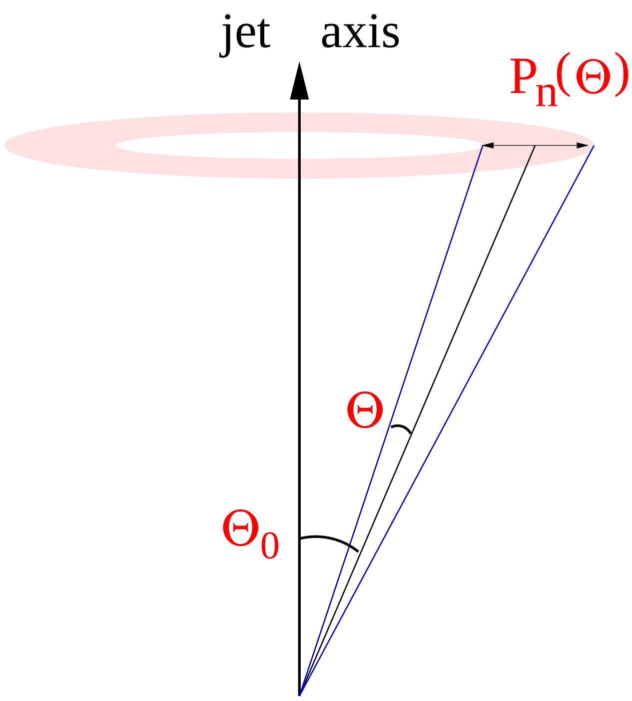
$$H_q = \frac{K_q}{F_q}$$

QCD (+ LPHD) analytic calculations (Dremin and Hwa) for H_q

$r_{H_q} = \frac{H_q(\mathrm{gluon})}{H_q(\mathrm{quark})}$	$q = 2$	$q = 3$
OPAL data (prel.)	$0.30 \pm 0.11 \pm 0.10$	$0.01 \pm 0.18 \pm 0.11$
JETSET, hemisph.	0.39 ± 0.01	0.07 ± 0.01
HERWIG, hemisph.	0.33 ± 0.01	0.13 ± 0.01
QCD ($\frac{1}{2}$ event)	0.51	0.23
JETSET, $\frac{1}{2}$ event	0.35 ± 0.01	0.07 ± 0.01
HERWIG, $\frac{1}{2}$ event	0.30 ± 0.01	0.09 ± 0.01

Data agree with analytic QCD (within large errors).

Angular Intermittancy



L3 study of Factorial moments in angular bins about sphericity axis

$P_n(\Theta)$ - prob. of detecting n hadrons at Θ

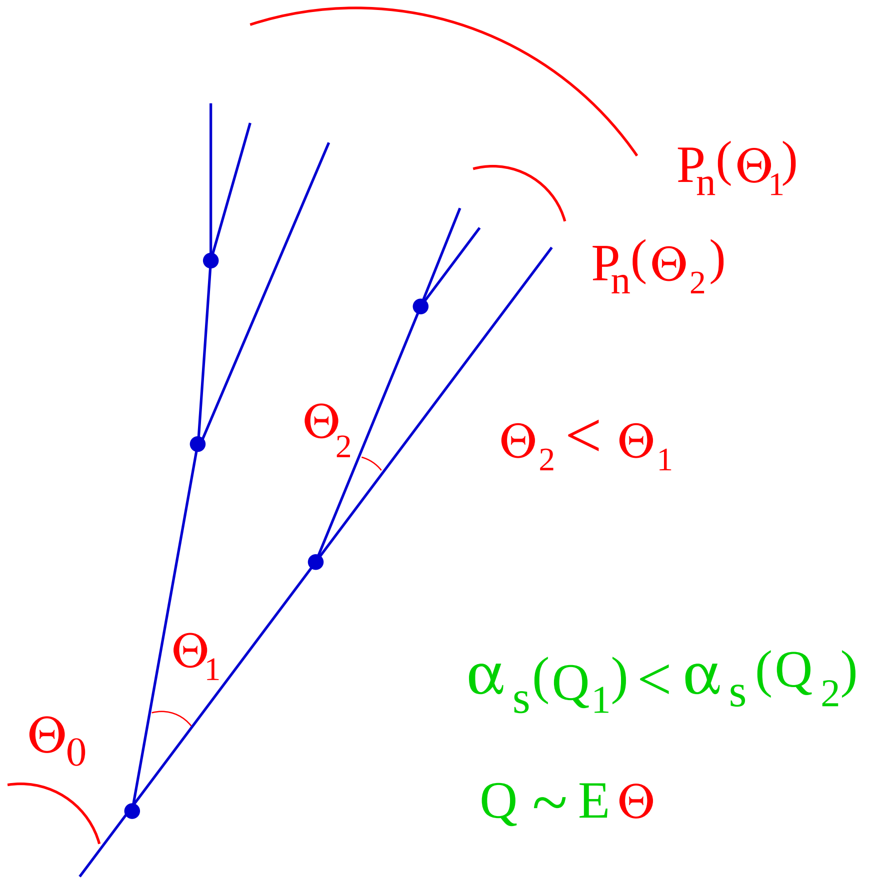
- $\Theta \quad \Rightarrow \quad z = \dfrac{\ln(\Theta/\Theta_0)}{\ln(E\Theta/\Lambda)} \qquad E \simeq \dfrac{\sqrt{s}}{2}$

  $0 < \Theta < \Theta_0 \quad \Rightarrow \quad \infty > z > 0$

- $P_n(\Theta) \quad \Rightarrow \quad P_n(z) \quad \rightarrow \quad F_q(z)$

  normalized factorial moments

Angular Intermittancy Test of QCD



QCD PREDICTIONS:

- Strong correlations of partons

$$F_q(\Theta) \propto \left(\frac{\Theta_0}{\Theta}\right)^{(1-D_q)(q-1)},$$

$$D_q \neq 1$$

- Dependence of D_q on α_s

Angular Intermittancy Test of QCD

- Fixed coupling regime in DLLA:

$$D_q = \gamma_0 \frac{q+1}{q},$$

$\gamma_0 = \sqrt{2\, C_{\mathrm{A}} \alpha_s / \pi}$.

- Running coupling regime in DLLA:

Yu.Dokshitzer, I.M.Dremin, Nucl.Phys. B402 (1993) 139

$$D_q \simeq \gamma_0 \frac{q+1}{q} \left(1 + \frac{q^2+1}{4q^2} z \right) \qquad (a)$$

- Another DLLA estimation:

Ph.Brax, J.-L.Meunier, R.Peschanski, Z.Phys. C62 (1994) 649

$$D_q \simeq 2\,\gamma_0 \frac{q+1}{q} \left(\frac{1-\sqrt{1-z}}{z} \right) \qquad (b)$$

- Running coupling regime in MLLA:

Yu.Dokshitzer, I.M.Dremin, Nucl.Phys. B402 (1993) 139

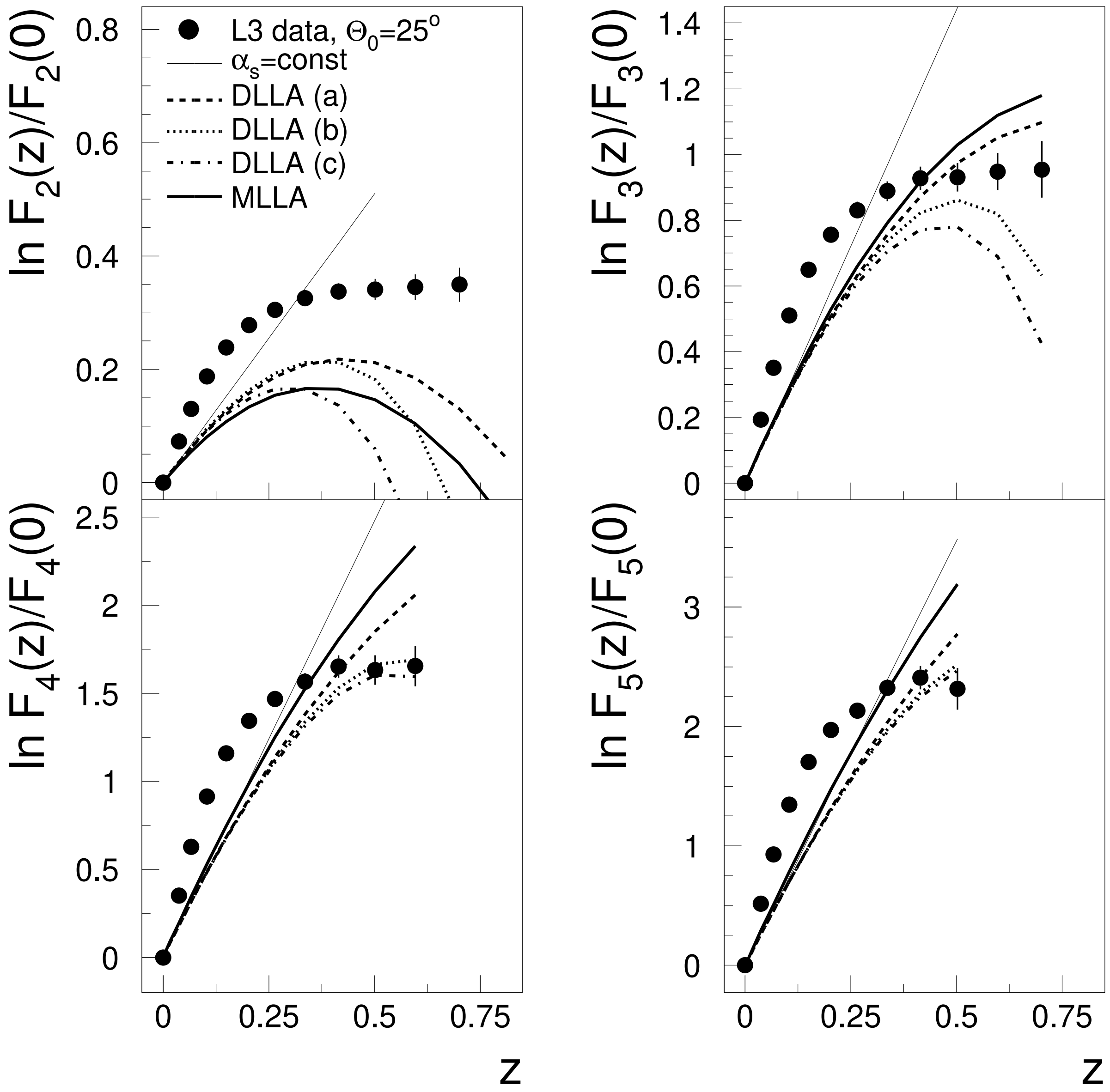
$$\gamma_0 \rightarrow \gamma_0^{\mathrm{eff}} = \gamma_0 + \gamma_0^2 \frac{b}{4C_{\mathrm{A}}} \left[-B \frac{q-1}{2(q+1)} + S_q \right]$$

$$S_q = \frac{q-1}{2(q+1)(q^2+1)} + \frac{1}{4}, \quad b = \frac{11C_{\mathrm{A}}}{3} - \frac{2n_f}{3}, \quad B = \frac{1}{b} \left[\frac{11C_{\mathrm{A}}}{3} + \frac{2n_f}{3C_{\mathrm{A}}^2} \right].$$

Angular Intermittancy Test of QCD

(preliminary)

$\Theta_0 = 25^\circ$ $\Lambda = 0.16\,\mathrm{GeV}$ $n_f = 3$

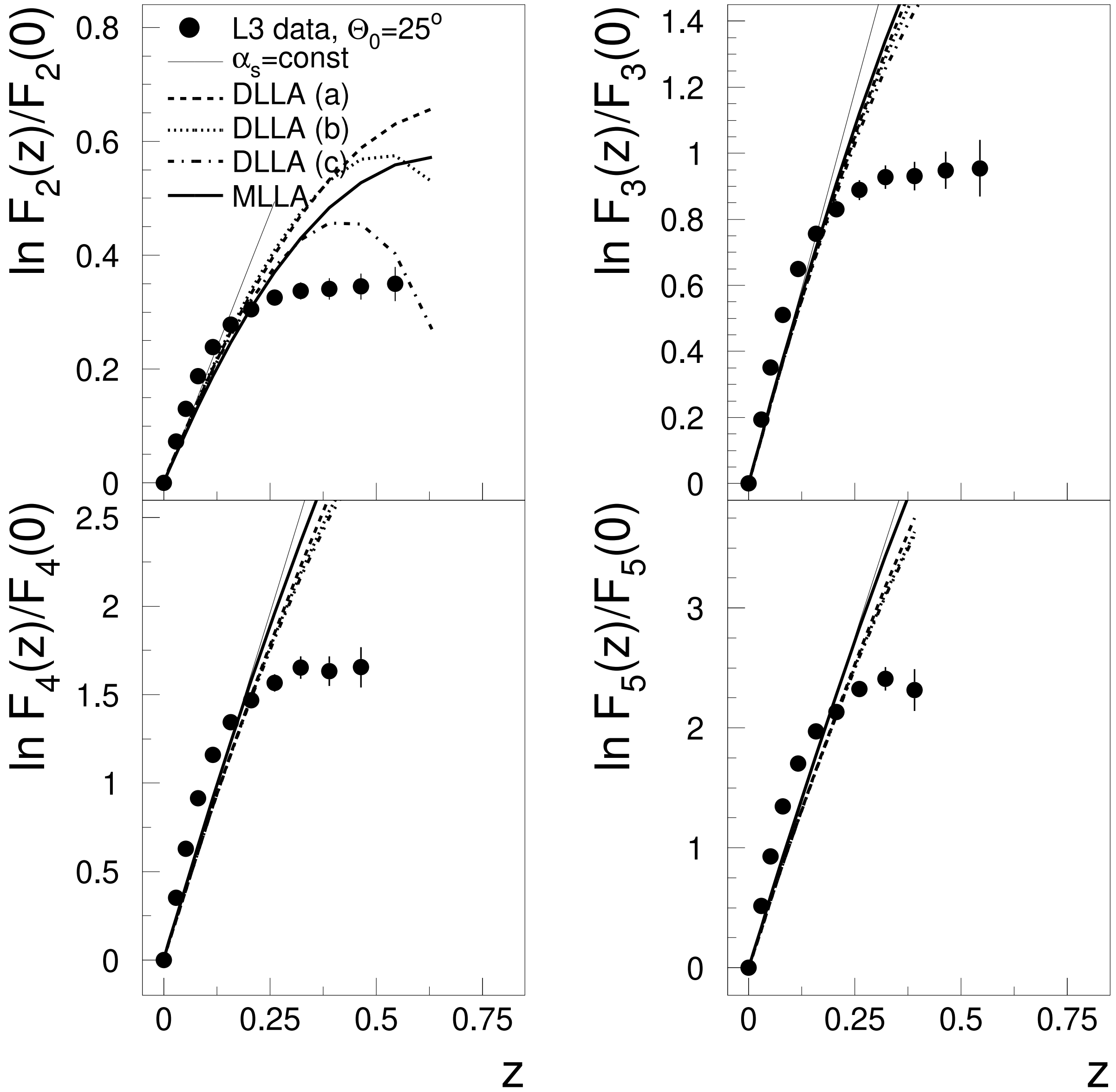


None of predictions agree with data

Angular Intermittancy Test of QCD

(preliminary)

$\Theta_0 = 25°$ $\Lambda = 0.04\,\mathrm{GeV}$ $n_f = 3$



Disagreement is less for smaller Λ
but still not good

Possible reasons (escape clause):

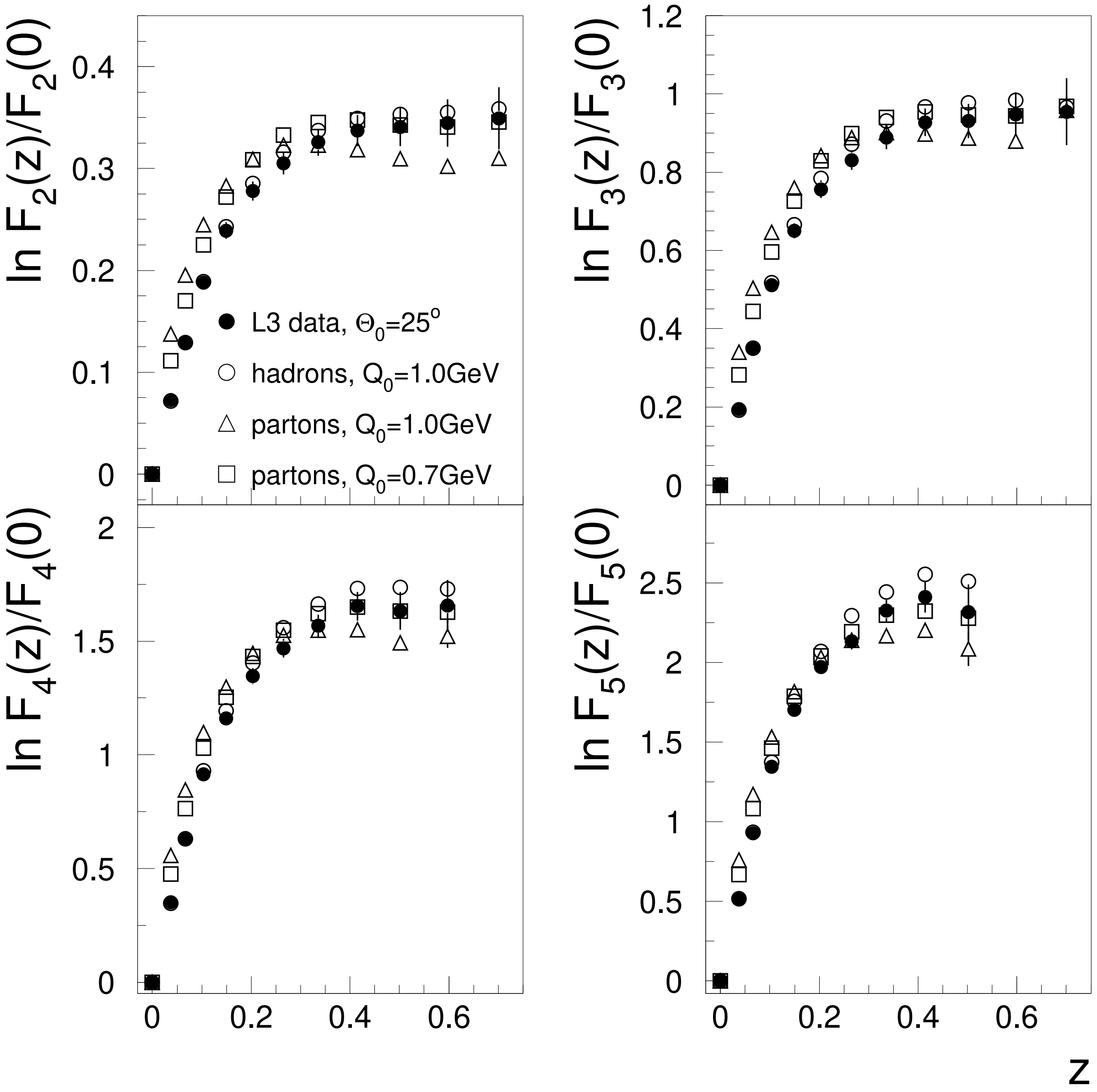
Need higher orders of pert. QCD

Non-pert. effects or invalidity of LPHD

Angular Intermittancy Test of QCD

(preliminary)

$\Theta_0 = 25^\circ$ JETSET



But JETSET agrees well

Analytical QCD + LPHD or models

- QCD calculations are for partons, not particles
  – can not calculate at such small scales (pion mass)

- to compare with data, must rely on
  - assumptions like LPHD
  - models like string fragmentation in JETSET

- So you do not really know if you are testing QCD or hadronization model or assumptions

Subjets

A possible way out:

- do not use particles
- but use jets (called subjets) defined at a scale
  - smaller than that used to classify events as 2-, 3-, 4-jet
  - large enough that QCD calculations still valid

Using Durham ($k_\perp$) jet algorithm,

$$y_{ij} = \frac{2\ \min(E_i^2, E_j^2)}{E_{\text{vis}}^2}(1 - \cos\theta_{ij})$$

- Combine particles i,j with smallest y_{ij} into new "particle"
- Repeat until no pair left with $y_{ij} < y_{\text{cut}}$
- Result is subjets at scale corresponding to y_{cut}
- $2\ \min(E_i^2, E_j^2)(1 - \cos\theta_{ij}) \approx p_{\text{t}}^2$ of particle i w.r.t. j (to second order in θ_{ij})
- p_{t}^2 is a reasonable choice of QCD scale Q^2
- Thus the energy scale of a subjet is $E_{\text{cms}}\sqrt{y_{\text{cut}}}$

For perturbative scales ($\gtrsim$ 1 GeV)
Mean Subjet Multiplicity for 2- and 3-jet events
agrees well with analytic QCD calculations
of the mean gluon multiplicity

For an example of this type of analysis see talk by
D. Mangeol in Parallel Session A

Subjets

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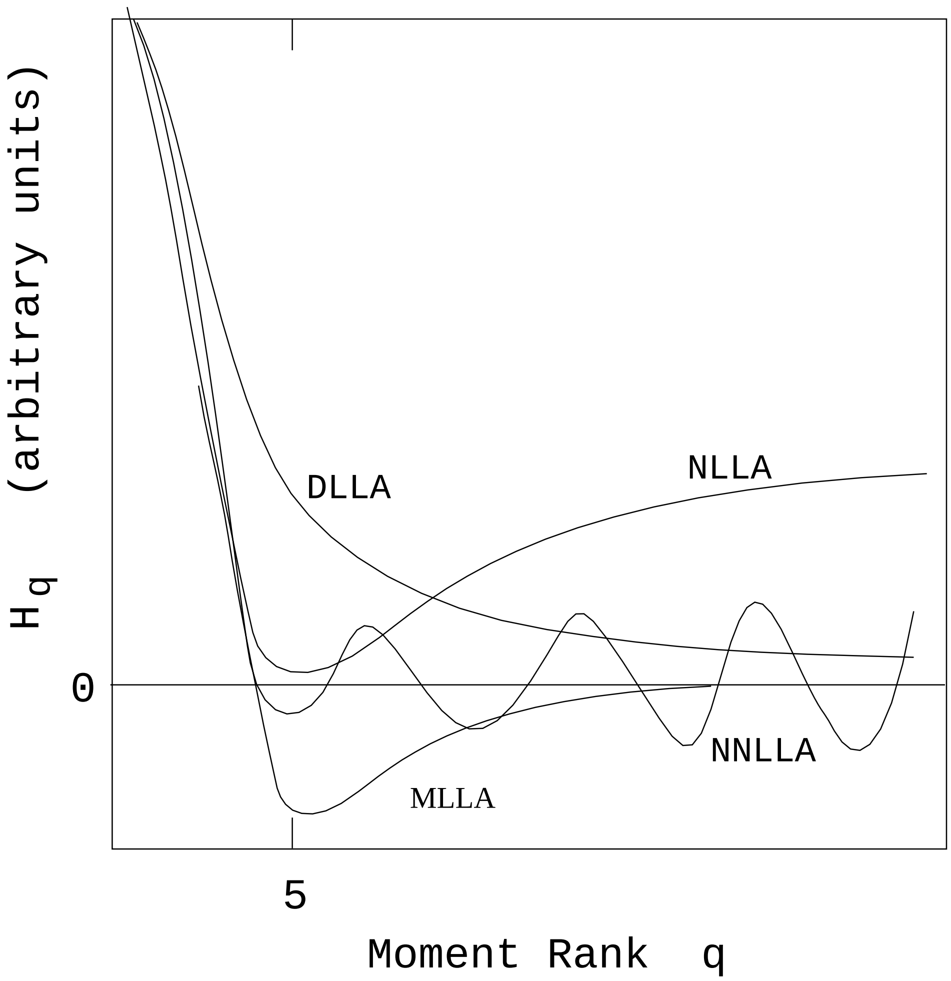
Charged Particle Multiplicity

Two years ago SLD analysed the mult. dist. in terms of $H_q = \frac{K_q}{F_q}$

QCD predicts behaviour of H_q

Assuming LPHD, same behaviour should occur for charged particle mult. dist.



SLD observed the behaviour predicted by NNLLA, *i.e.*, min. + oscillations

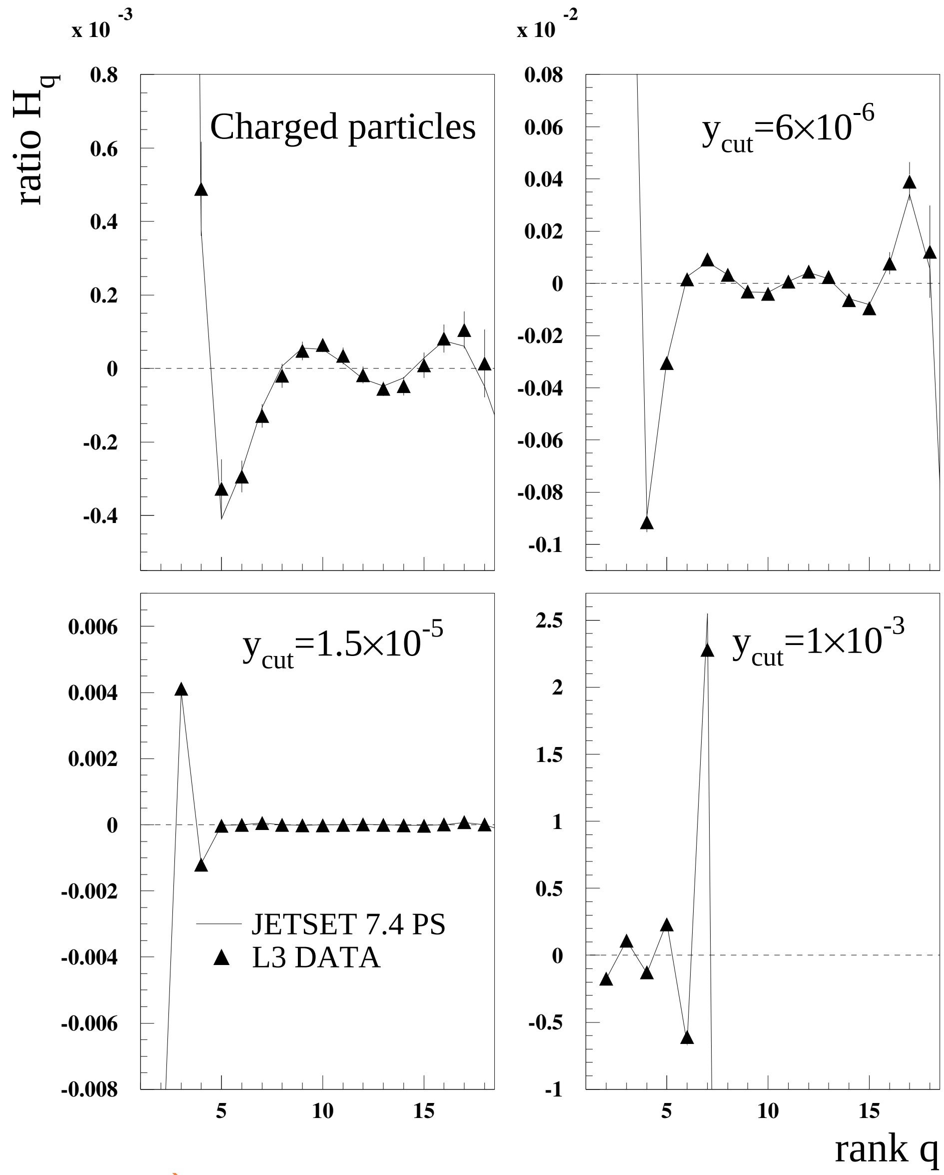
Charged Particle Multiplicity

This behaviour confirmed by L3

But also seen in JETSET, which is only MLLA

Could behaviour be due to something else?

Look at subjets, jets for small y_{cut} (Durham)



As y_{cut} (E scale) increases, min. + oscillations disappear long before pert. region (where prediction should be best)

Observed min. + oscillations seems merely fortuituous,

NOT a confirmation of NNLLA

Conclusions

- Basic assumptions of QCD are confirmed
  vector gluons, SU(3), α_s runs

- QCD works well where calculations are most reliable

- as scale decreases
  - calculations become less reliable
  - assumptions or models become necessary
  - you are not always sure what you are testing

- Subjets offer a way of separating perturbative effects from the effects of non-perturbative hadronization

- We have not falsified QCD (yet).

- But maybe because of escape clause:
  higher-order terms

These transparencies can be found in
`http://wwwinfo.cern.ch/~metzger/talks.html`