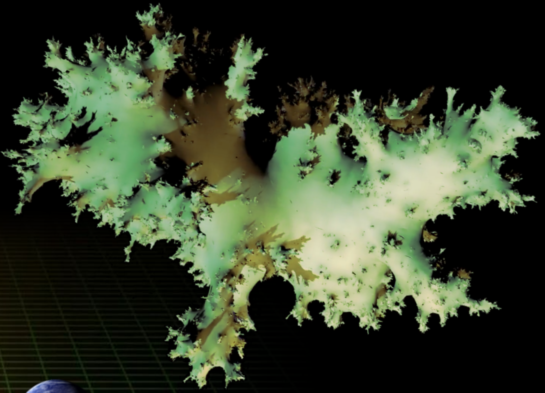
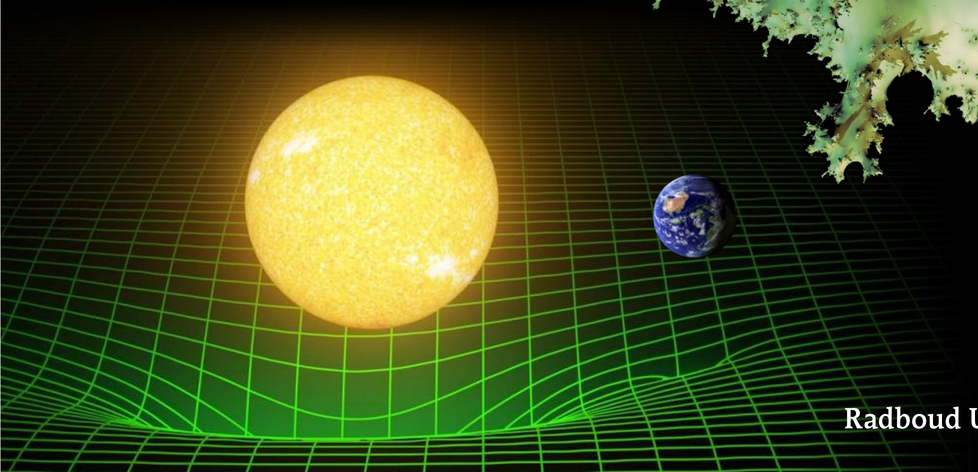
3rd EPS conference on Gravitation ~ 23-05-2022

# Quantum Gravity & Random Geometry


Timothy Budd




Radboud University

# Renormalization in quantum gravity

- How to make sense of the formal gravitational path integral?

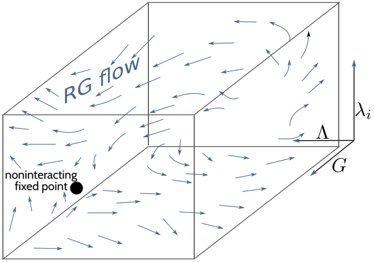
$$Z_{\mathrm{Lor}} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}} e^{\frac{i}{\hbar}S[g]} \xleftrightarrow{\text{Wick?}} Z_{\mathrm{Eucl}} = \int_M \frac{\mathcal{D}g_{ab}}{\mathrm{Diff}} e^{-S[g]}$$

# Renormalization in quantum gravity

- How to make sense of the formal gravitational path integral?

$$Z_{\mathrm{Lor}} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}} e^{\frac{i}{\hbar}S[g]} \overset{\text{Wick?}}{\longleftrightarrow} Z_{\mathrm{Eucl}} = \int_M \frac{\mathcal{D}g_{ab}}{\mathrm{Diff}} e^{-S[g]}$$

- As usual in QFT: coupling constants are subject to renormalization.



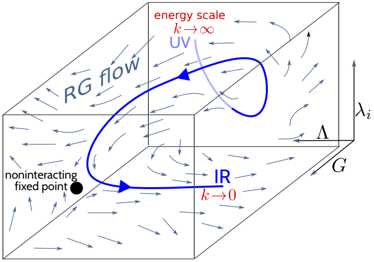
Theory space $\{G, \Lambda, \lambda_1, \lambda_2, \ldots\}$

# Renormalization in quantum gravity

- How to make sense of the formal gravitational path integral?

$$Z_{\mathrm{Lor}} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}} e^{\frac{i}{\hbar}S[g]} \overset{\text{Wick?}}{\longleftrightarrow} Z_{\mathrm{Eucl}} = \int_M \frac{\mathcal{D}g_{ab}}{\mathrm{Diff}} e^{-S[g]}$$

- As usual in QFT: coupling constants are subject to renormalization.



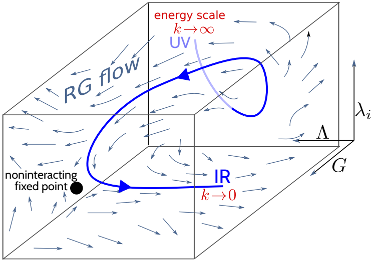
Theory space $\{G, \Lambda, \lambda_1, \lambda_2, \ldots\}$

# Renormalization in quantum gravity

- How to make sense of the formal gravitational path integral?

$$Z_{\mathrm{Lor}} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}} e^{\frac{i}{\hbar}S[g]} \overset{\text{Wick?}}{\longleftrightarrow} Z_{\mathrm{Eucl}} = \int_M \frac{\mathcal{D}g_{ab}}{\mathrm{Diff}} e^{-S[g]}$$

- As usual in QFT: coupling constants are subject to renormalization.
- Perturbation theory is non-renormalizable: couplings proliferate in the UV.



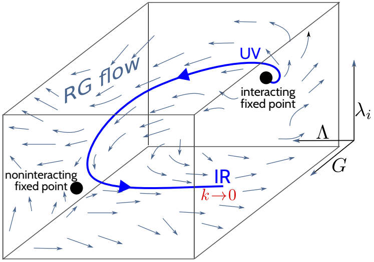
Theory space $\{G, \Lambda, \lambda_1, \lambda_2, \ldots\}$

# Renormalization in quantum gravity

- How to make sense of the formal gravitational path integral?

$$Z_{\mathrm{Lor}} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}}\, e^{\frac{i}{\hbar}S[g]} \overset{\text{Wick?}}{\longleftrightarrow} Z_{\mathrm{Eucl}} = \int_M \frac{\mathcal{D}g_{ab}}{\mathrm{Diff}}\, e^{-S[g]}$$

- As usual in QFT: coupling constants are subject to renormalization.
- Perturbation theory is non-renormalizable: couplings proliferate in the UV.
- Asymptotic safety hypothesis: UV completeness and predictivity can be restored if RG has an interacting fixed point. [Weinberg, Wilson, ...]



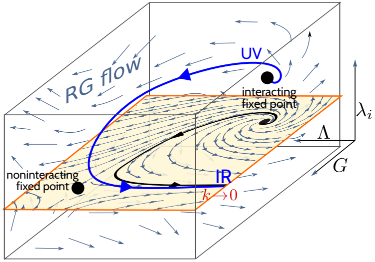
Theory space $\{G, \Lambda, \lambda_1, \lambda_2, \ldots\}$

# Renormalization in quantum gravity

- How to make sense of the formal gravitational path integral?

$$Z_{\rm Lor} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\rm Diff}\, e^{\frac{i}{\hbar}S[g]} \overset{\text{Wick?}}{\longleftrightarrow} Z_{\rm Eucl} = \int_M \frac{\mathcal{D}g_{ab}}{\rm Diff}\, e^{-S[g]}$$

- As usual in QFT: coupling constants are subject to renormalization.
- Perturbation theory is non-renormalizable: couplings proliferate in the UV.
- Asymptotic safety hypothesis: UV completeness and predictivity can be restored if RG has an interacting fixed point. [Weinberg, Wilson, . . . ]
- Evidence for existence of suitable Reuter fixed point from Functional Renormalization Group methods. Percacci's talk! [Reuter, Lauscher, Saueressig, . . . ]



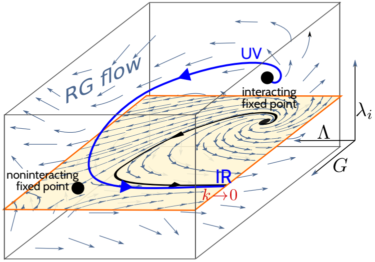
Theory space $\{G, \Lambda, \lambda_1, \lambda_2, \ldots\}$

# Renormalization in quantum gravity

- How to make sense of the formal gravitational path integral?

$$Z_{\mathrm{Lor}} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}} e^{\frac{i}{\hbar}S[g]} \overset{\text{Wick?}}{\longleftrightarrow} Z_{\mathrm{Eucl}} = \int_M \frac{\mathcal{D}g_{ab}}{\mathrm{Diff}} e^{-S[g]}$$

- As usual in QFT: coupling constants are subject to renormalization.
- Perturbation theory is non-renormalizable: couplings proliferate in the UV.
- Asymptotic safety hypothesis: UV completeness and predictivity can be restored if RG has an interacting fixed point. [Weinberg, Wilson, . . . ]
- Evidence for existence of suitable Reuter fixed point from Functional Renormalization Group methods. Percacci's talk! [Reuter, Lauscher, Saueressig, . . . ]
- Physics at fixed point in $Z_{\mathrm{Eucl}}$: scale-invariant random geometry.



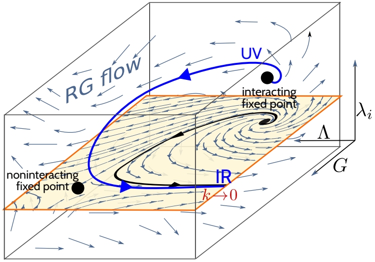

Theory space $\{G, \Lambda, \lambda_1, \lambda_2, \ldots\}$

# Renormalization in quantum gravity

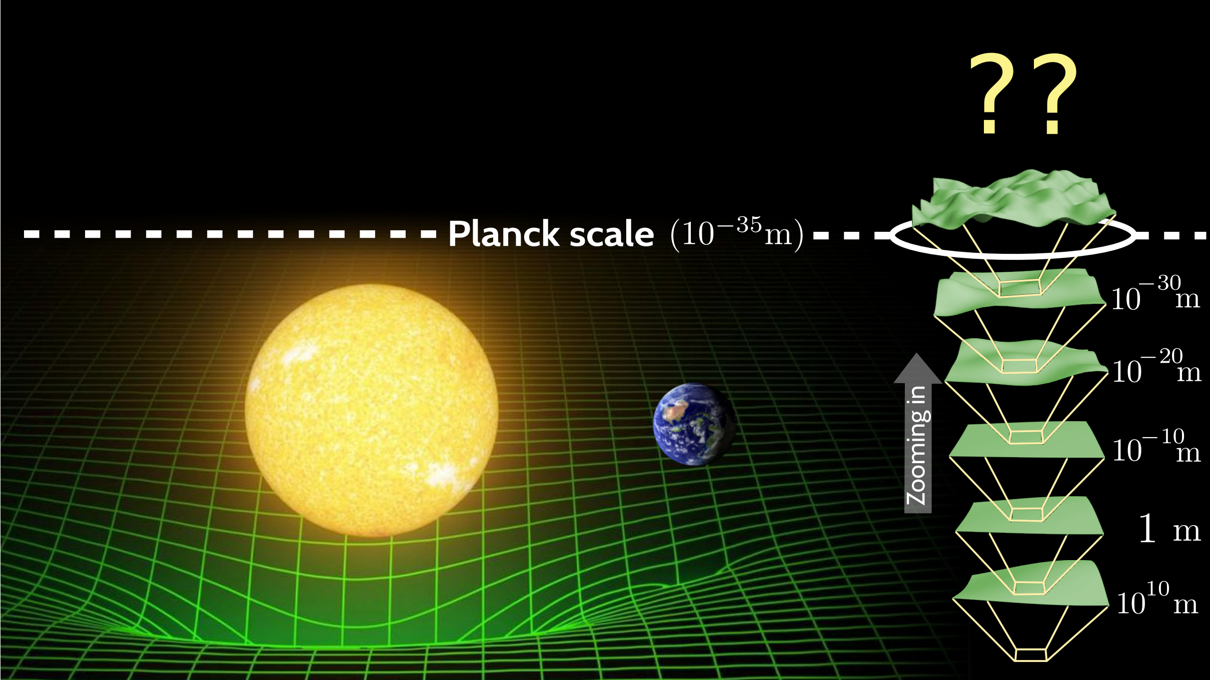
- How to make sense of the formal gravitational path integral?

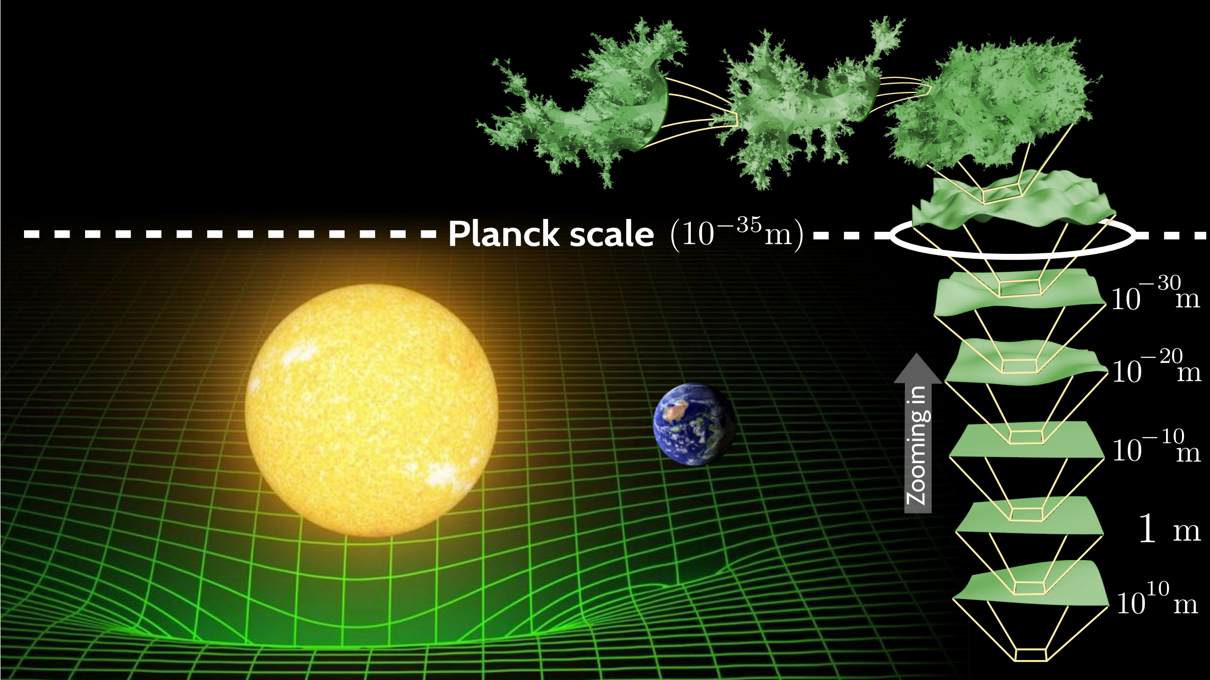
$$Z_{\mathrm{Lor}} = \int_M \frac{\mathcal{D}g_{\mu\nu}}{\mathrm{Diff}} e^{\frac{i}{\hbar}S[g]} \overset{\text{Wick?}}{\longleftrightarrow} Z_{\mathrm{Eucl}} = \int_M \frac{\mathcal{D}g_{ab}}{\mathrm{Diff}} e^{-S[g]}$$

- As usual in QFT: coupling constants are subject to renormalization.
- Perturbation theory is non-renormalizable: couplings proliferate in the UV.
- Asymptotic safety hypothesis: UV completeness and predictivity can be restored if RG has an interacting fixed point. [Weinberg, Wilson, . . . ]
- Evidence for existence of suitable Reuter fixed point from Functional Renormalization Group methods. Percacci's talk! [Reuter, Lauscher, Saueressig, . . . ]
- Physics at fixed point in $Z_{\mathrm{Eucl}}$: scale-invariant random geometry.



Theory space $\{G, \Lambda, \lambda_1, \lambda_2, \ldots\}$


??
Planck scale $(10^{-35}\mathrm{m})$
$10^{-30}\mathrm{m}$
$10^{-20}\mathrm{m}$
$10^{-10}\mathrm{m}$
$1\ \mathrm{m}$
$10^{10}\mathrm{m}$
Zooming in



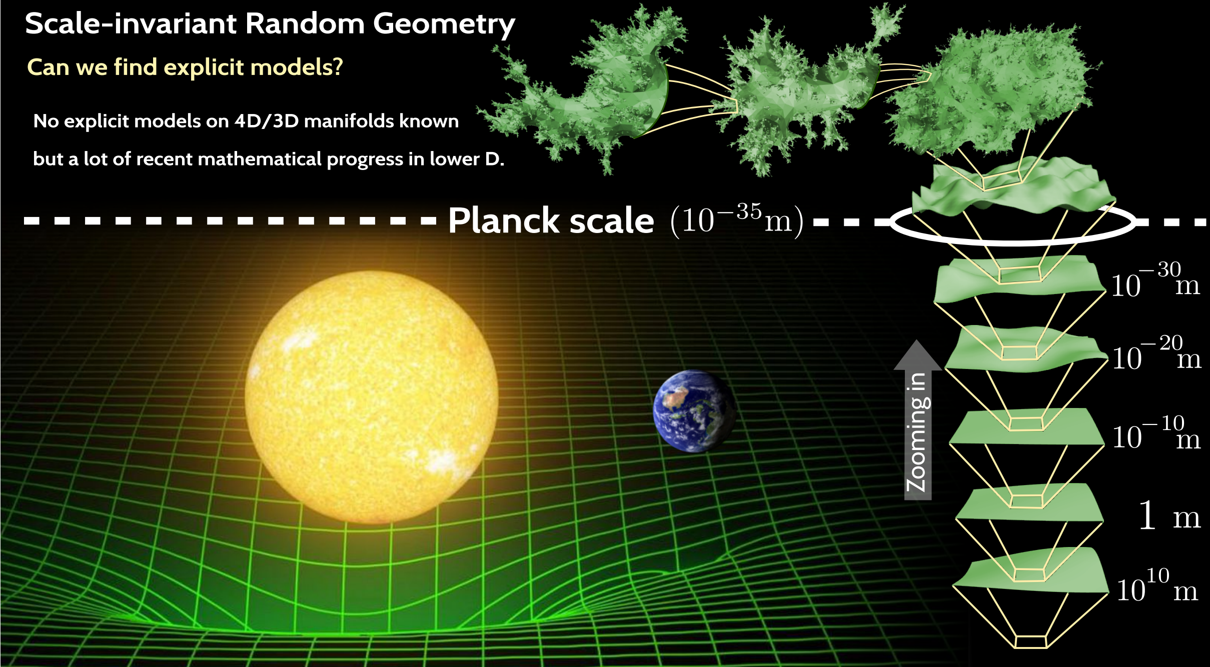
Planck scale $(10^{-35}\mathrm{m})$
$10^{-30}\mathrm{m}$
$10^{-20}\mathrm{m}$
Zooming in
$10^{-10}\mathrm{m}$
$1\ \mathrm{m}$
$10^{10}\mathrm{m}$

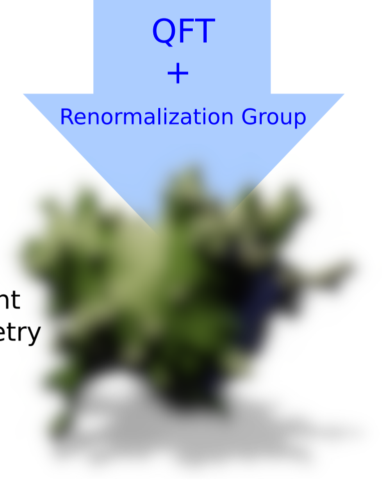
# Scale-invariant Random Geometry

## Can we find explicit models?

No explicit models on 4D/3D manifolds known

but a lot of recent mathematical progress in lower D.





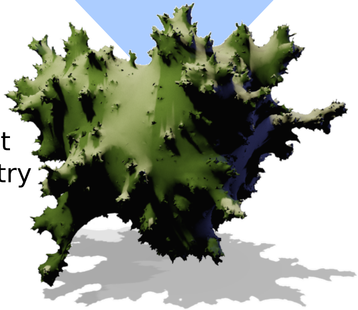

scale-invariant
random geometry

QFT

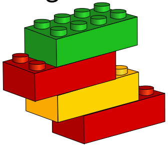
+

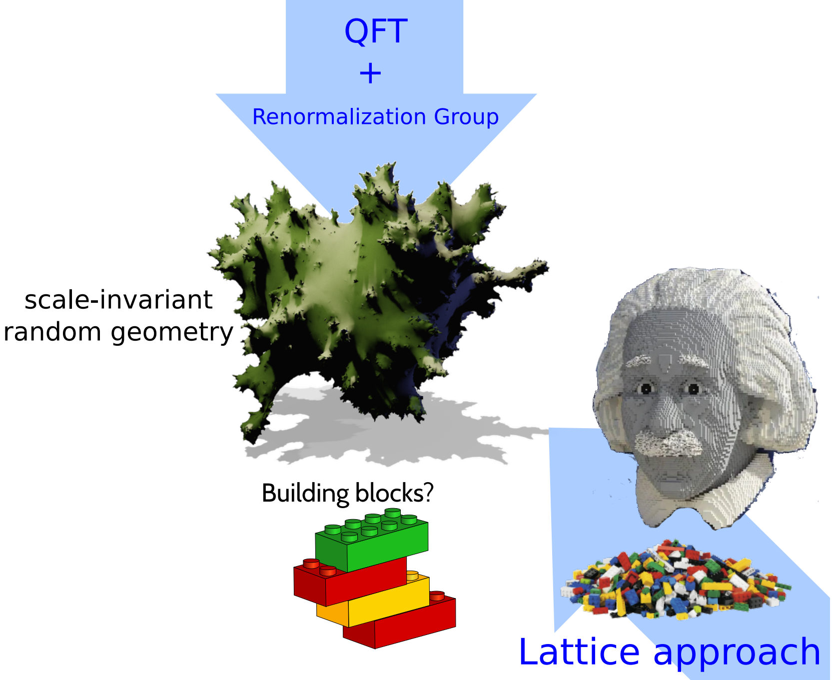
Renormalization Group

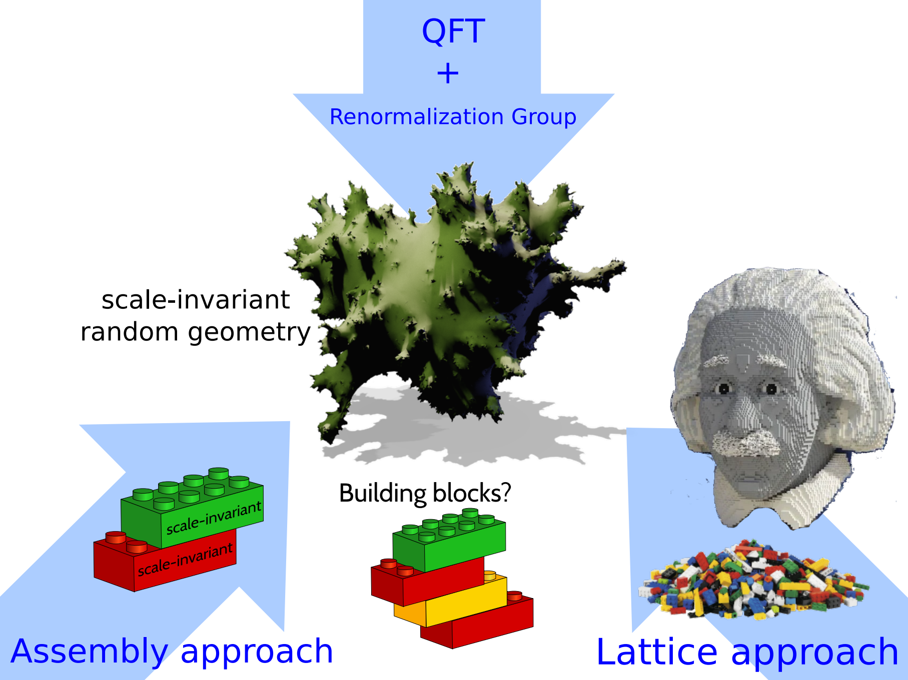
scale-invariant
random geometry



Building blocks?

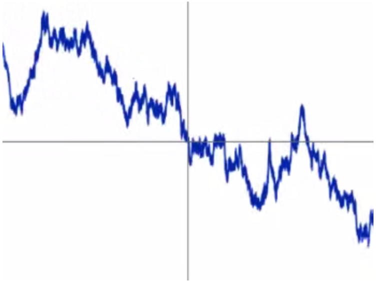



QFT
+
Renormalization Group
scale-invariant
random geometry
Building blocks?
Lattice approach


QFT
+
Renormalization Group
scale-invariant
random geometry
scale-invariant
scale-invariant
Building blocks?
Assembly approach
Lattice approach

# The simplest universality class

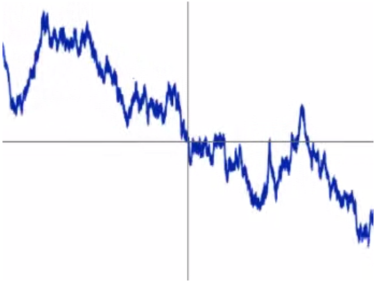
- The most familiar scale-invariant object?
  Brownian motion $B_t,\ t \in \mathbb{R}$ $\quad\equiv\quad$ 1D real scalar field



# The simplest universality class

- The most familiar scale-invariant object?
  Brownian motion $B_t$, $t \in \mathbb{R}$ $\equiv$ 1D real scalar field
- Exactly self-similar (in statistical sense):

$$\sqrt{\lambda} B_{t/\lambda} \overset{\text{law}}{=} B_t .$$

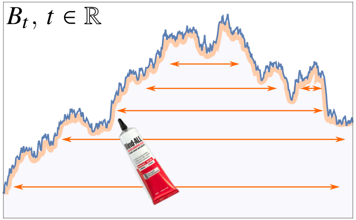


# The simplest universality class

- The most familiar scale-invariant object?
  Brownian motion $B_t,\ t \in \mathbb{R}$ $\equiv$ 1D real scalar field
- Exactly self-similar (in statistical sense):

$$\sqrt{\lambda} B_{t/\lambda} \overset{\text{law}}{=} B_t.$$

- Not 'geometry', but can assemble it into one:
  Continuum Random Tree [Aldous, '91]
  $\equiv$ Branched-polymer universality class

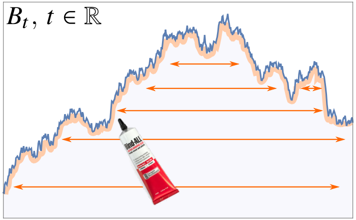


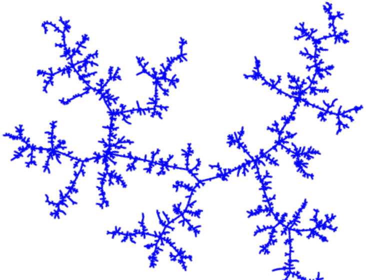
# The simplest universality class

- The most familiar scale-invariant object?
  Brownian motion $B_t,\ t \in \mathbb{R}$ $\quad\equiv\quad$ 1D real scalar field
- Exactly self-similar (in statistical sense):

$$\sqrt{\lambda} B_{t/\lambda} \stackrel{\text{law}}{=} B_t .$$

- Not 'geometry', but can assemble it into one:
  Continuum Random Tree [Aldous, '91]
  $\equiv$ Branched-polymer universality class



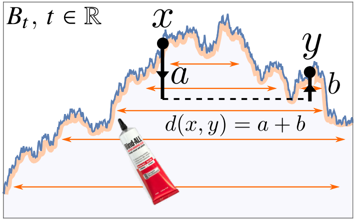



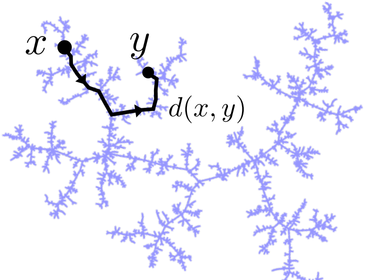
# The simplest universality class

- The most familiar scale-invariant object?
  Brownian motion $B_t$, $t \in \mathbb{R}$ $\equiv$ 1D real scalar field
- Exactly self-similar (in statistical sense):

$$\sqrt{\lambda} B_{t/\lambda} \overset{\text{law}}{=} B_t.$$

- Not ‘geometry’, but can assemble it into one:
  Continuum Random Tree [Aldous, ’91]
  $\equiv$ Branched-polymer universality class



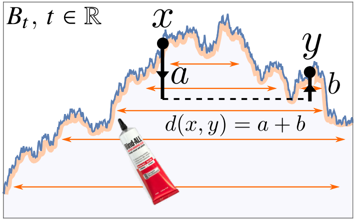



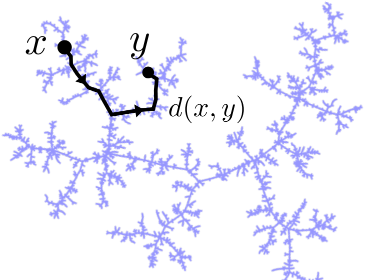
# The simplest universality class

- The most familiar scale-invariant object?
  Brownian motion $B_t,\ t \in \mathbb{R}$ $\equiv$ 1D real scalar field
- Exactly self-similar (in statistical sense):

$$\sqrt{\lambda} B_{t/\lambda} \overset{\text{law}}{=} B_t.$$

- Not 'geometry', but can assemble it into one:
  Continuum Random Tree [Aldous, '91]
  $\equiv$ Branched-polymer universality class
- Well-defined (geodesic) distances but genuinely fractal:
  - Topological dimension: 1
  - Hausdorff dimension: 2 $(V \sim r^2)$
  - Spectral dimension: $4/3$



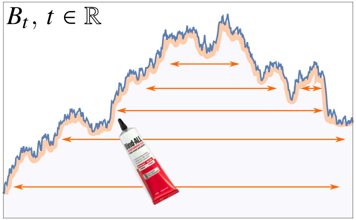



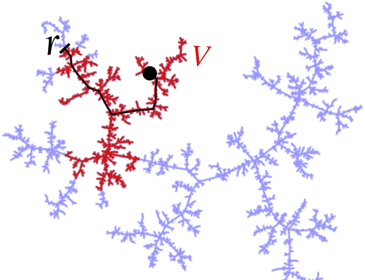
# The simplest universality class

- The most familiar scale-invariant object?
  Brownian motion $B_t,\ t \in \mathbb{R}$ $\quad\equiv\quad$ 1D real scalar field
- Exactly self-similar (in statistical sense):

$$\sqrt{\lambda} B_{t/\lambda} \quad \overset{\text{law}}{=} \quad B_t.$$

- Not ‘geometry’, but can assemble it into one:
  Continuum Random Tree [Aldous, ’91]
  $\equiv$ Branched-polymer universality class
- Well-defined (geodesic) distances but genuinely fractal:
  - Topological dimension: 1
  - Hausdorff dimension: 2 $\quad (V \sim r^2)$
  - Spectral dimension: $4/3$



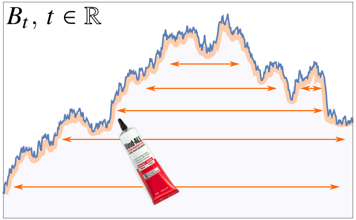



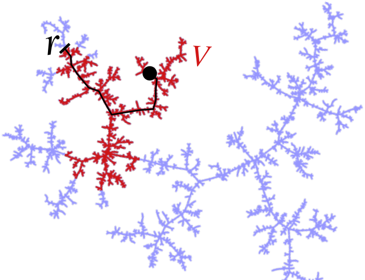
# The simplest universality class

- The most familiar scale-invariant object?
  Brownian motion $B_t,\ t \in \mathbb{R}$ $\quad\equiv\quad$ 1D real scalar field
- Exactly self-similar (in statistical sense):

$$\sqrt{\lambda} B_{t/\lambda} \stackrel{\text{law}}{=} B_t .$$

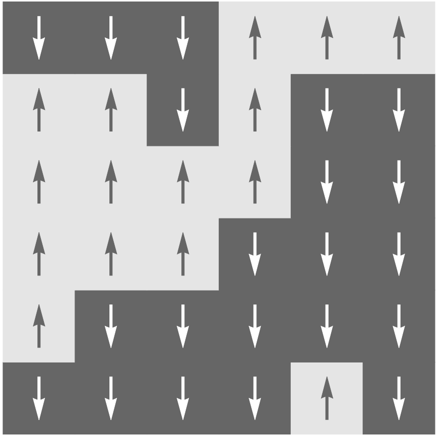
- Not 'geometry', but can assemble it into one:
  Continuum Random Tree [Aldous, '91]
  $\equiv$ Branched-polymer universality class
- Well-defined (geodesic) distances but genuinely fractal:
  - Topological dimension: 1
  - Hausdorff dimension: 2 $\quad (V \sim r^2)$
  - Spectral dimension: $4/3$
- Nothing like a spacetime geometry / manifold?!!



# Scale invariance and critical phenomena

Scale invariance also occurs in critical lattice models: for example the 2D Ising model at $T = T_c$.

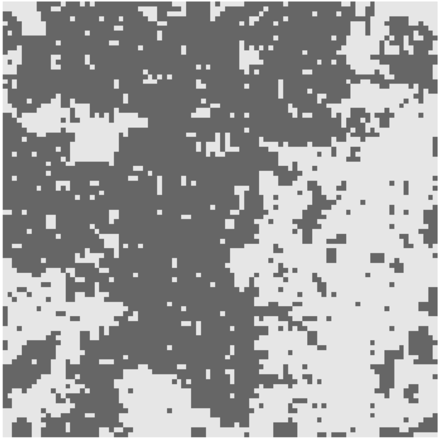


# Scale invariance and critical phenomena

Scale invariance also occurs in critical lattice models: for example the 2D Ising model at $T = T_c$.

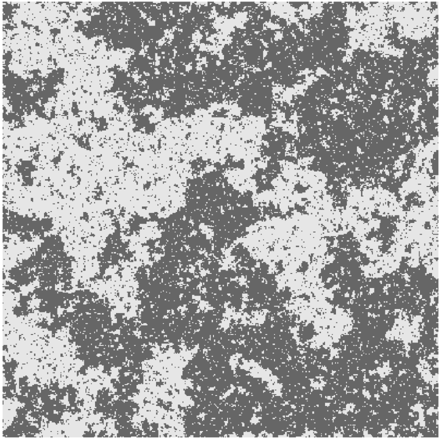
# Scale invariance and critical phenomena

Scale invariance also occurs in critical lattice models: for example the 2D Ising model at $T = T_c$.

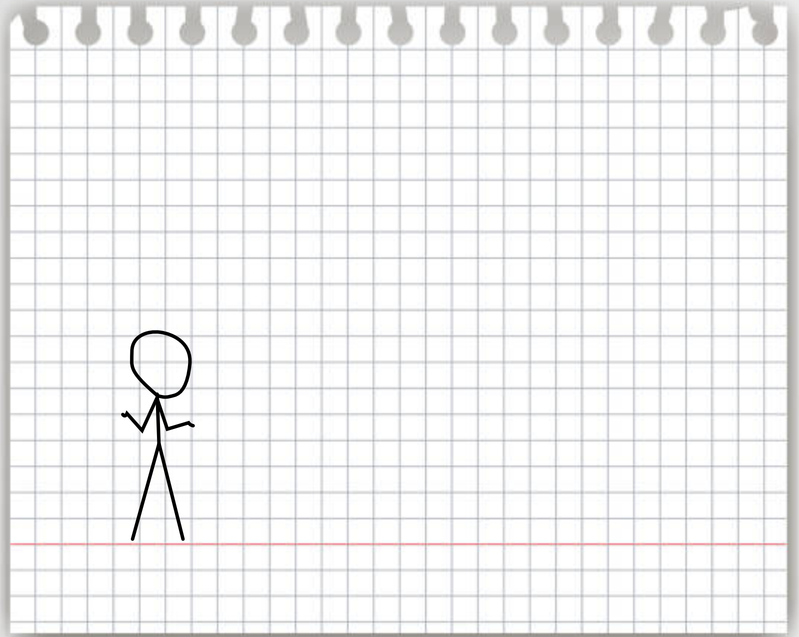


# Scale invariance and critical phenomena

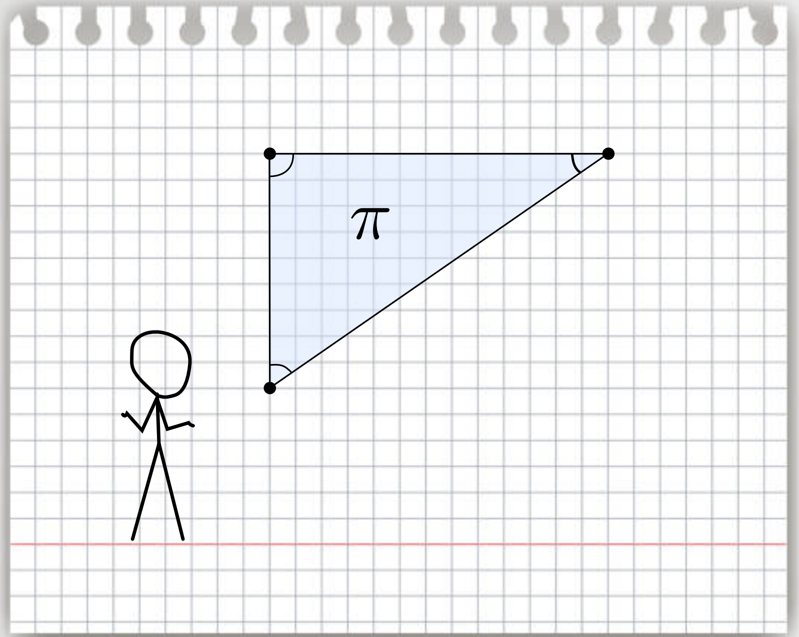
Scale invariance also occurs in critical lattice models: for example the 2D Ising model at $T = T_c$.



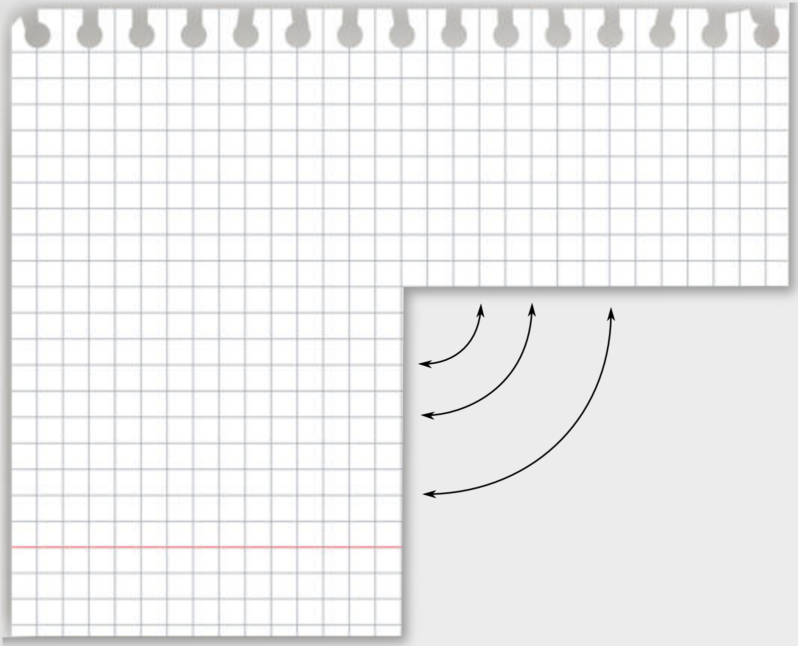
# 2D



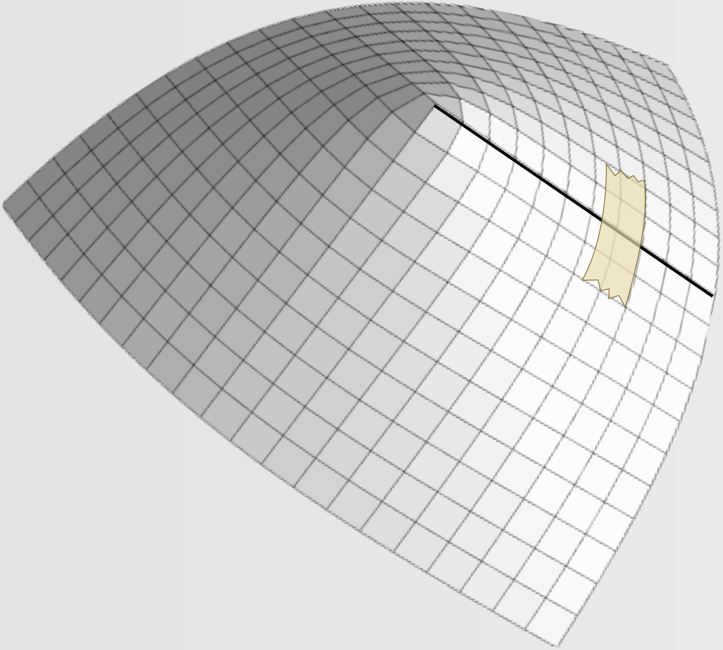
# 2D



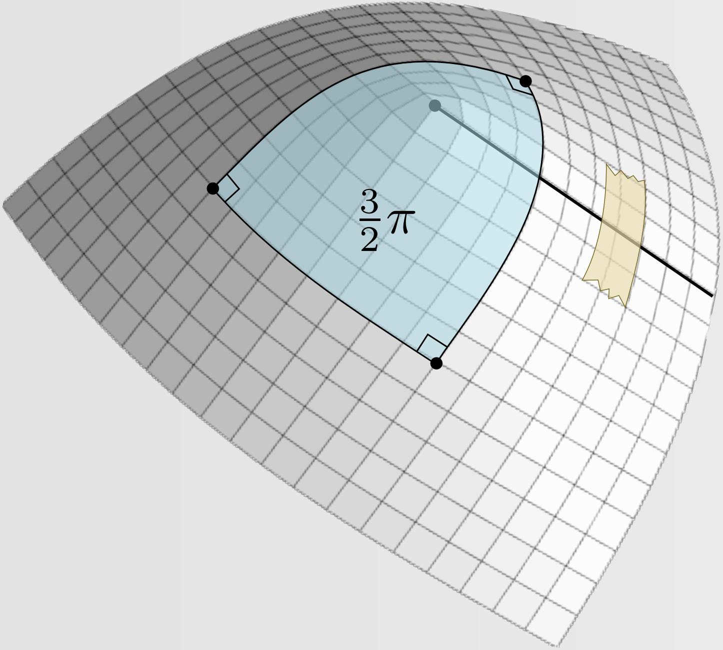
# 2D

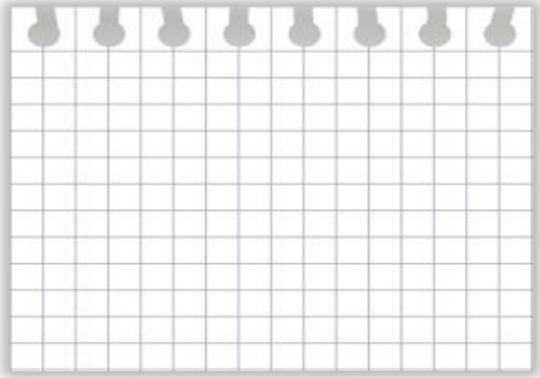


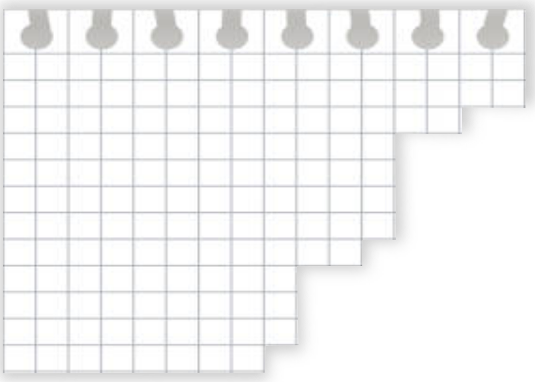
# 2D

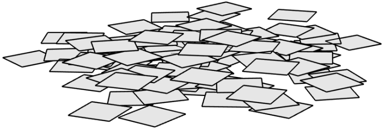


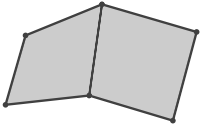
2D

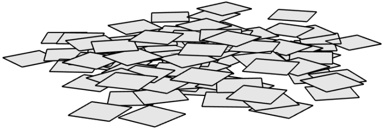


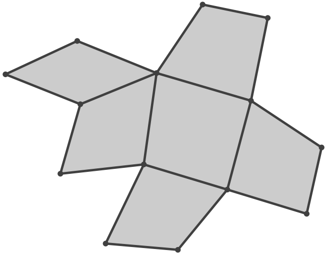


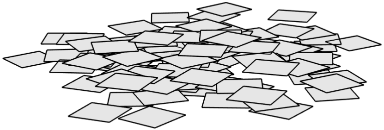


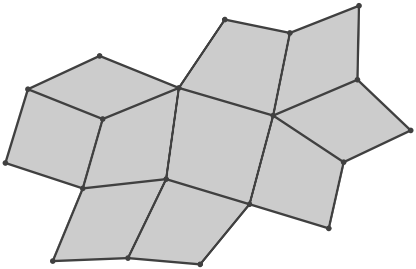


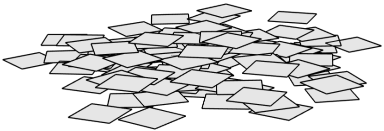


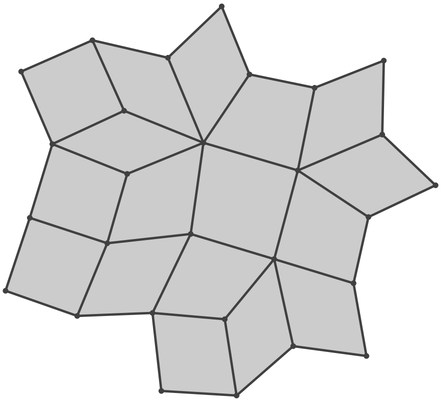


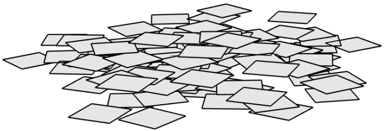


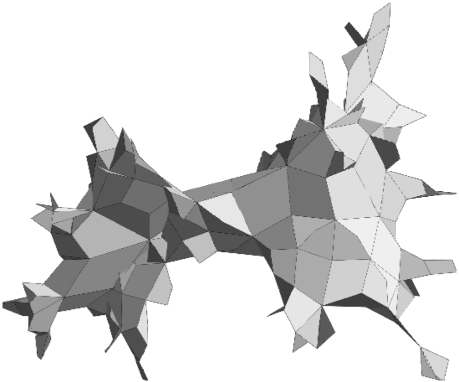


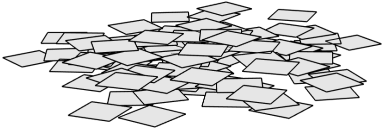




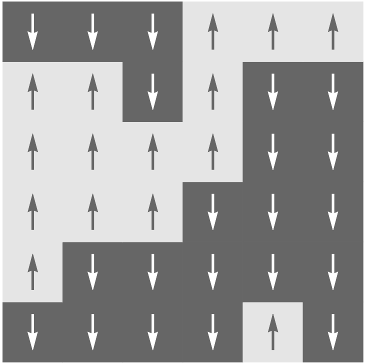




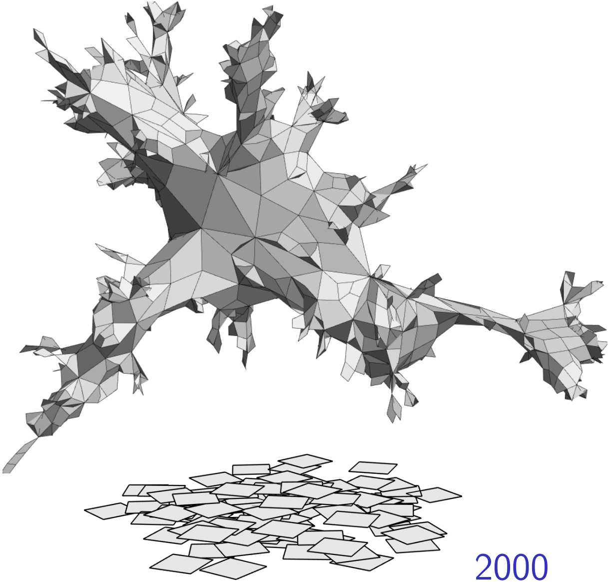




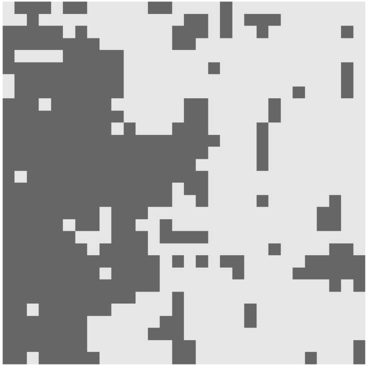
200



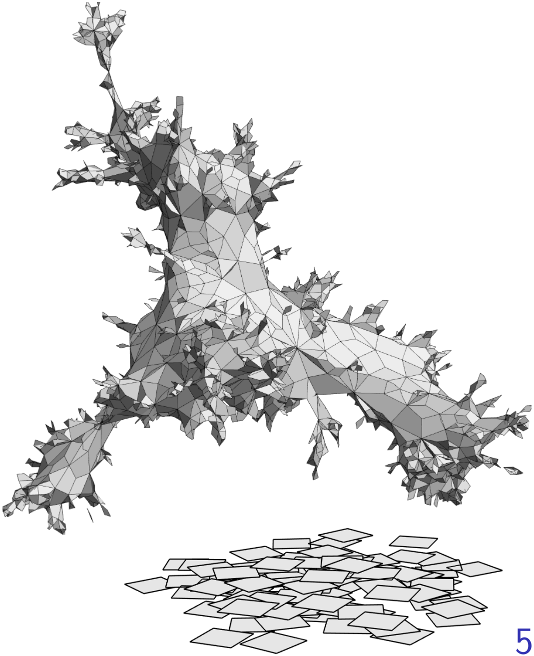
Ising model



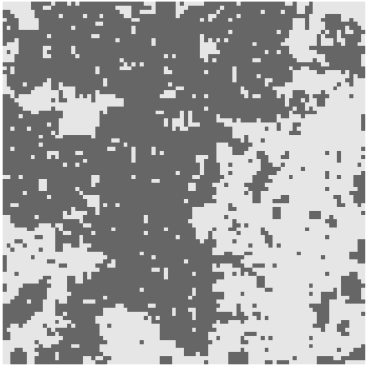
2000



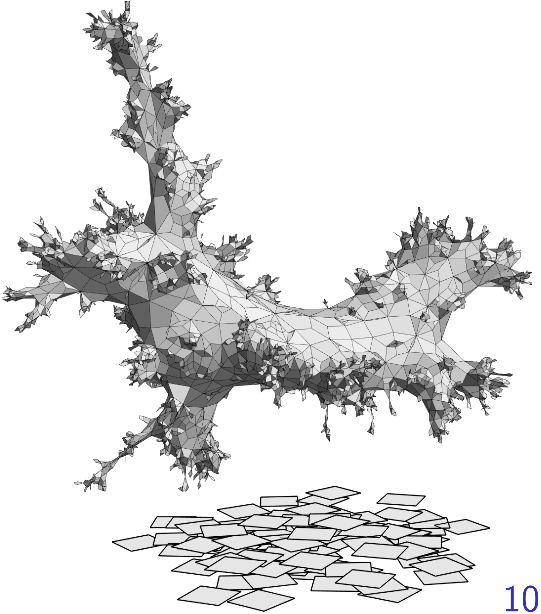
Ising model



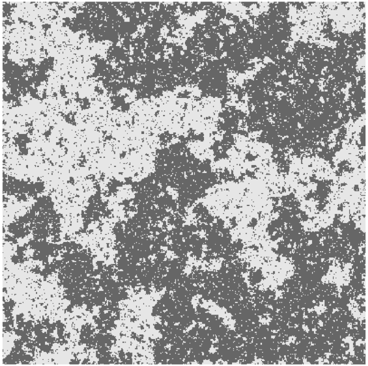
5000



Ising model

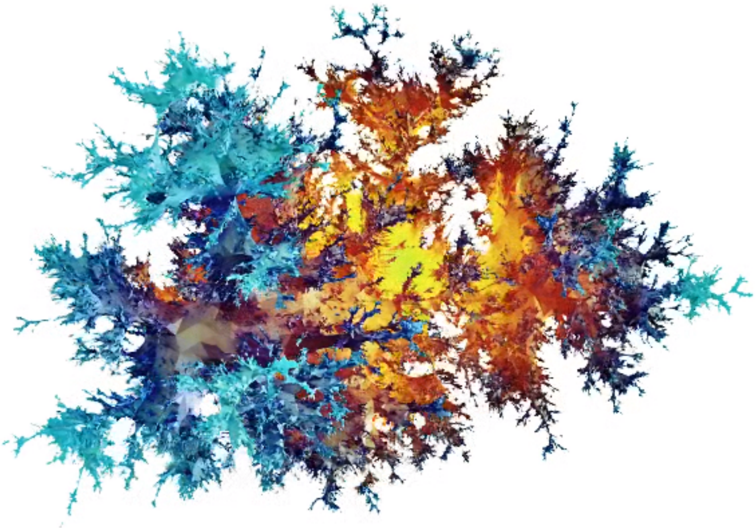


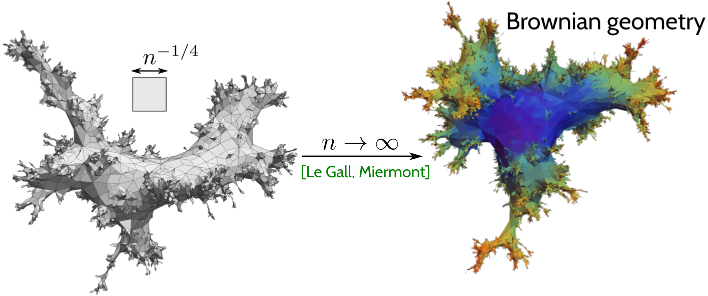
10000



Ising model

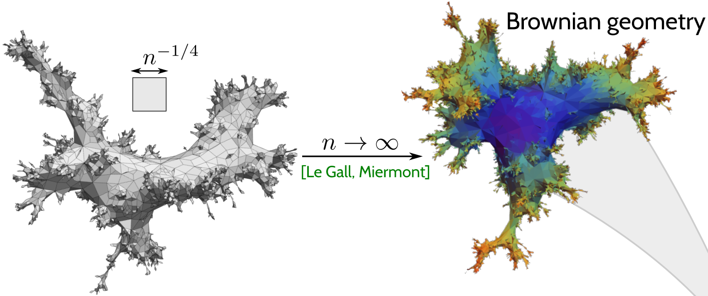
# Uniform quadrangulation (1 Million squares) [credits: B. Stufler]






- New universality class:
  *Brownian geometry* on 2-sphere

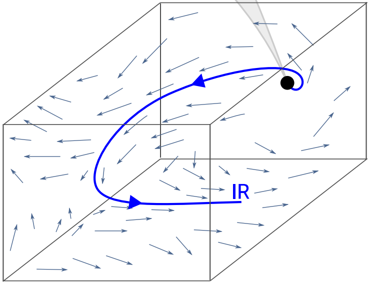
  *Topological* dimension: 2
  *Hausdorff* dimension: 4
  *Spectral* dimension: 2



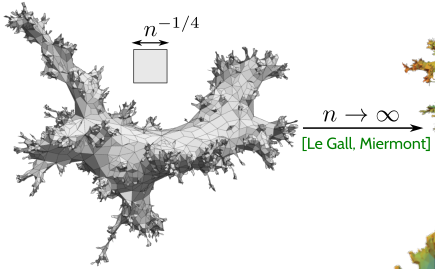

- New universality class:
  *Brownian geometry* on 2-sphere

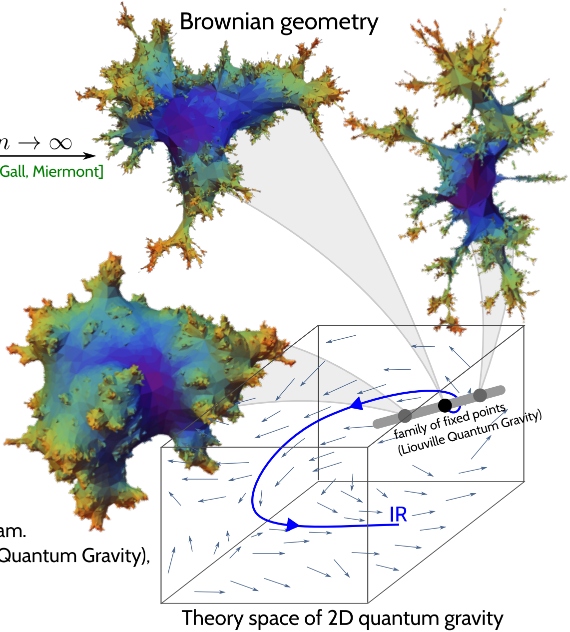
  *Topological* dimension: 2
  *Hausdorff* dimension: 4
  *Spectral* dimension: 2

- This *is* the UV fixed point of
  2D (Euclidean) Quantum Gravity!




Theory space of 2D quantum gravity



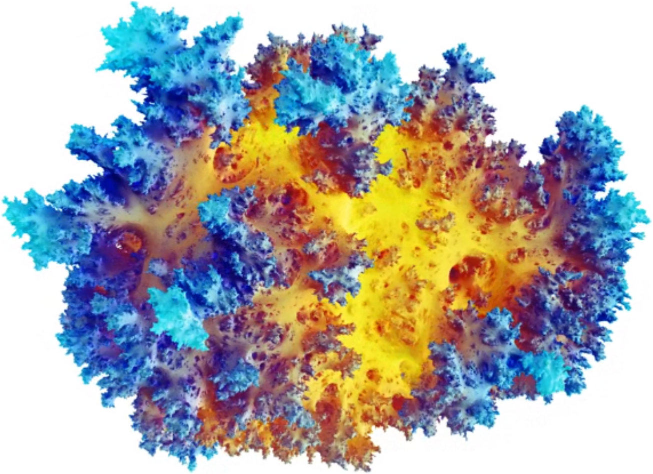

- New universality class:
  *Brownian geometry* on 2-sphere

  *Topological* dimension: 2
  *Hausdorff* dimension: 4
  *Spectral* dimension: 2

- This *is* the UV fixed point of
  2D (Euclidean) Quantum Gravity!

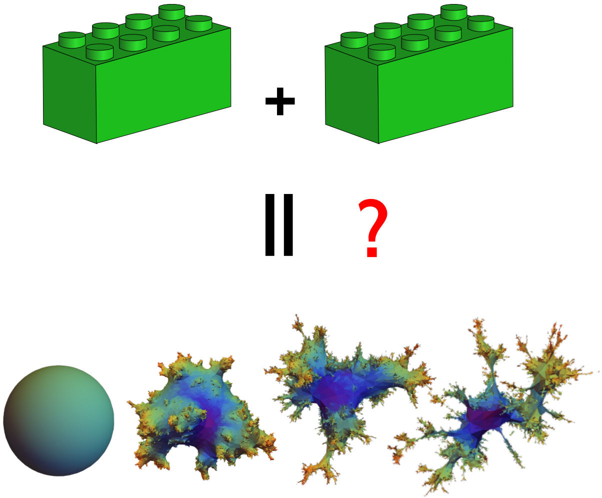
- Matter interaction extends it to a 1-param.
  family of universality classes (Liouville Quantum Gravity),
  with varying fractal dimensions.

# Coupling to matter (Schnyder wood): different universality class



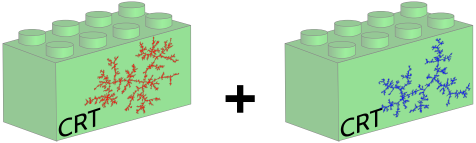
[credits: B. Stufler]

# Can 2D quantum gravity also be assembled?



Brownian sphere

# Can 2D quantum gravity also be assembled?

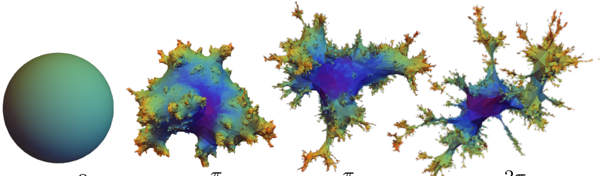



=

correlation angle
$\alpha \in (0, \pi)$

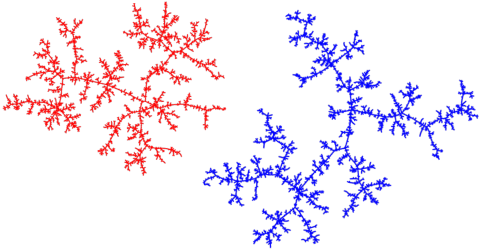
"Mating of trees"

[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]



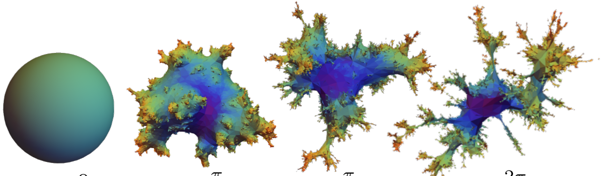
$\alpha \to 0$ $\alpha = \frac{\pi}{3}$ $\alpha = \frac{\pi}{2}$ $\alpha = \frac{2\pi}{3}$
Brownian sphere

# Can 2D quantum gravity also be assembled?



"Mating of trees"

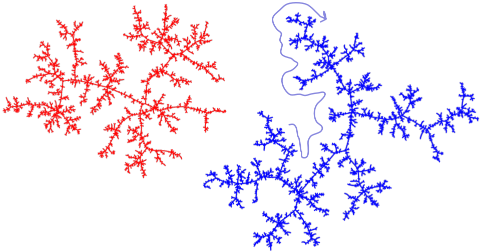
[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]

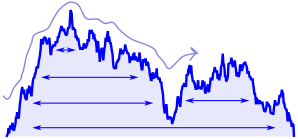


$\alpha \to 0$ $\alpha = \frac{\pi}{3}$ $\alpha = \frac{\pi}{2}$ $\alpha = \frac{2\pi}{3}$

Brownian sphere

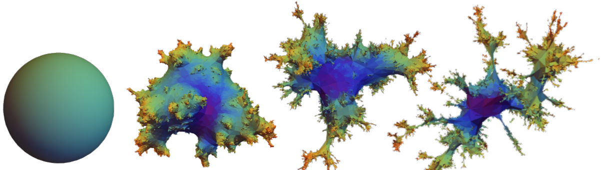
# Can 2D quantum gravity also be assembled?





"Mating of trees"

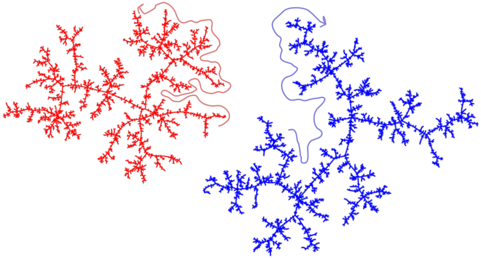
[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]

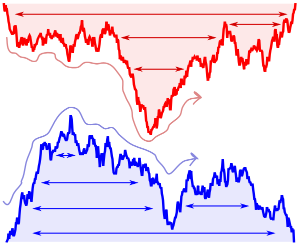


$\alpha \to 0$ $\alpha = \frac{\pi}{3}$ $\alpha = \frac{\pi}{2}$ $\alpha = \frac{2\pi}{3}$

Brownian sphere

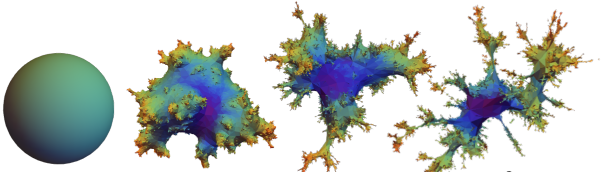
# Can 2D quantum gravity also be assembled?





"Mating of trees"

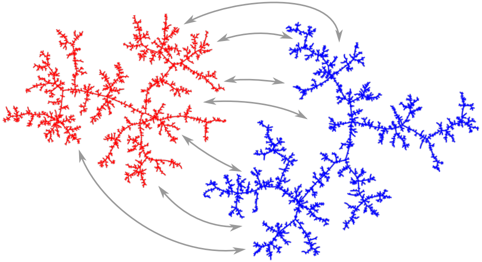
[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]

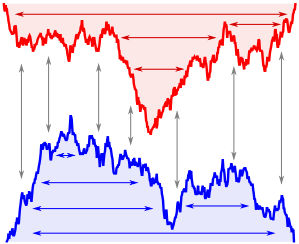


$\alpha \to 0$ $\alpha = \frac{\pi}{3}$ $\alpha = \frac{\pi}{2}$ $\alpha = \frac{2\pi}{3}$

Brownian sphere

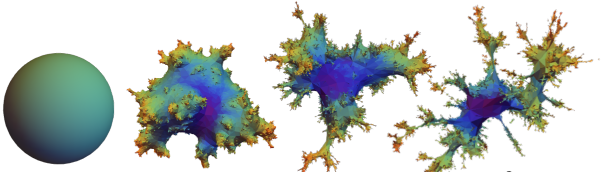
# Can 2D quantum gravity also be assembled?





"Mating of trees"

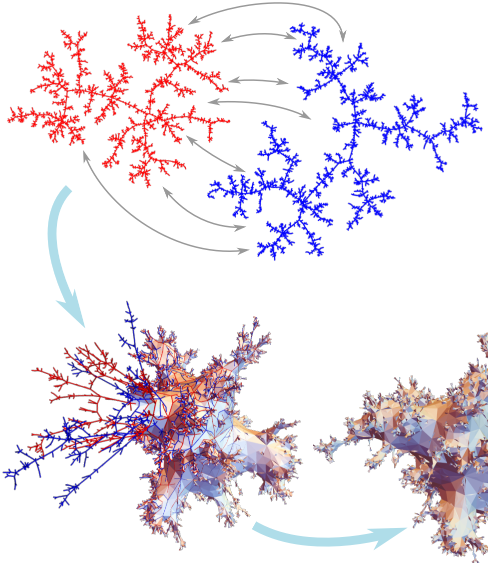
[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]

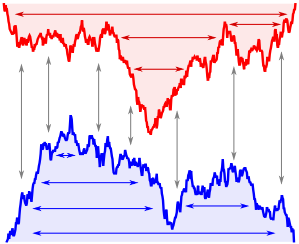


$\alpha \to 0$ $\alpha = \frac{\pi}{3}$ $\alpha = \frac{\pi}{2}$ $\alpha = \frac{2\pi}{3}$

Brownian sphere

# Can 2D quantum gravity also be assembled?



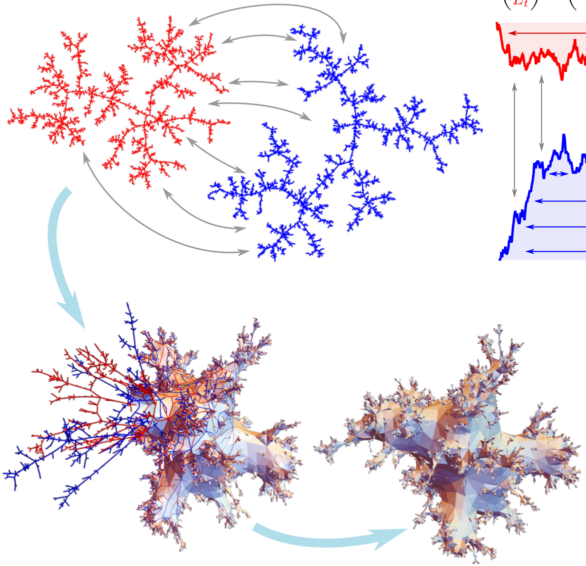


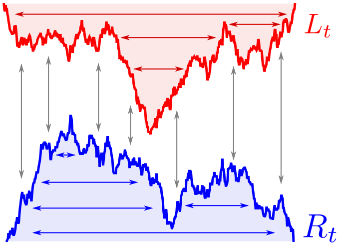
"Mating of trees"

[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]

# Can 2D quantum gravity also be assembled?

$$\begin{pmatrix} R_t \\ L_t \end{pmatrix} = \begin{pmatrix} \sin(\alpha) & -\cos(\alpha) \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} B_t^{(1)} \\ B_t^{(2)} \end{pmatrix} \quad \alpha \in (0, \pi)$$



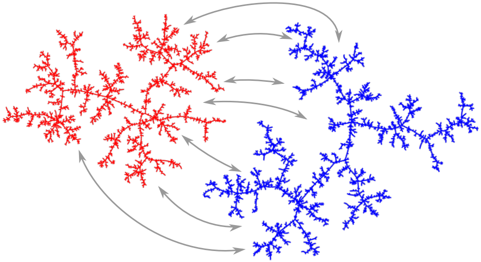


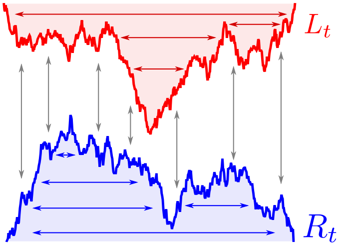

"Mating of trees"

[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]

# Can 2D quantum gravity also be assembled?

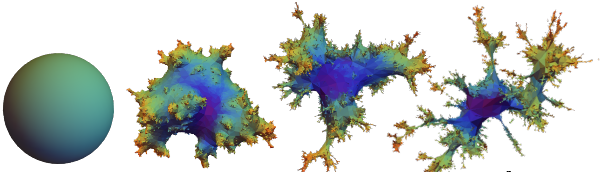
$$\begin{pmatrix} R_t \\ L_t \end{pmatrix} = \begin{pmatrix} \sin(\alpha) & -\cos(\alpha) \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} B_t^{(1)} \\ B_t^{(2)} \end{pmatrix} \quad \alpha \in (0, \pi)$$






"Mating of trees"

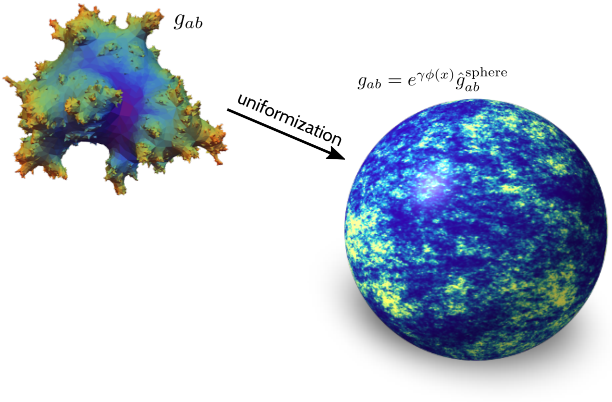
[Duplantier, Miller, Sheffield, '14]
[Gwynne, Holden, Sun, Bernardi,
Kenyon, Ding, Pfeffer, Kassel, Wilson, ...]



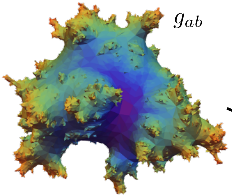
$\alpha \to 0$ $\quad$ $\alpha = \frac{\pi}{3}$ $\quad$ $\alpha = \frac{\pi}{2}$ $\quad$ $\alpha = \frac{2\pi}{3}$

Brownian sphere

# Why does this work?



# Why does this work?



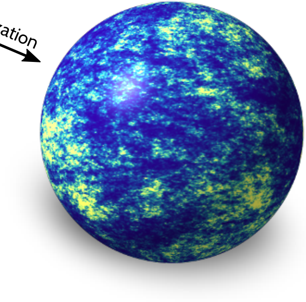
$g_{ab}$

uniformization

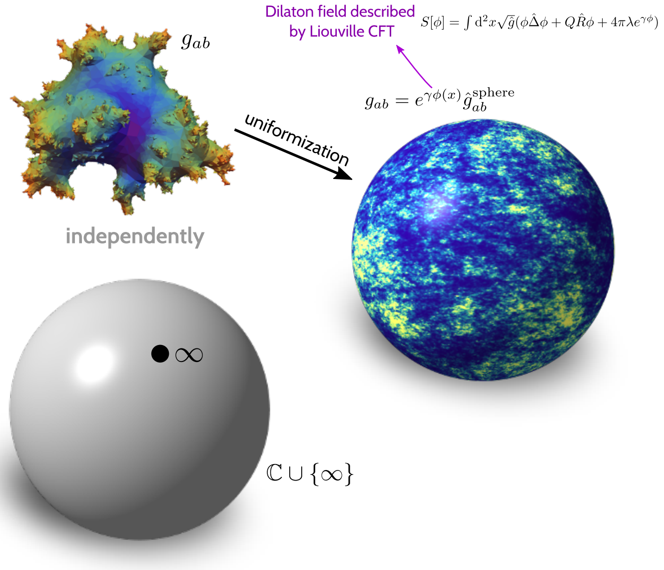
Dilaton field described by Liouville CFT

$$S[\phi] = \int \mathrm{d}^2x\sqrt{\hat{g}}(\phi\hat{\Delta}\phi + Q\hat{R}\phi + 4\pi\lambda e^{\gamma\phi})$$

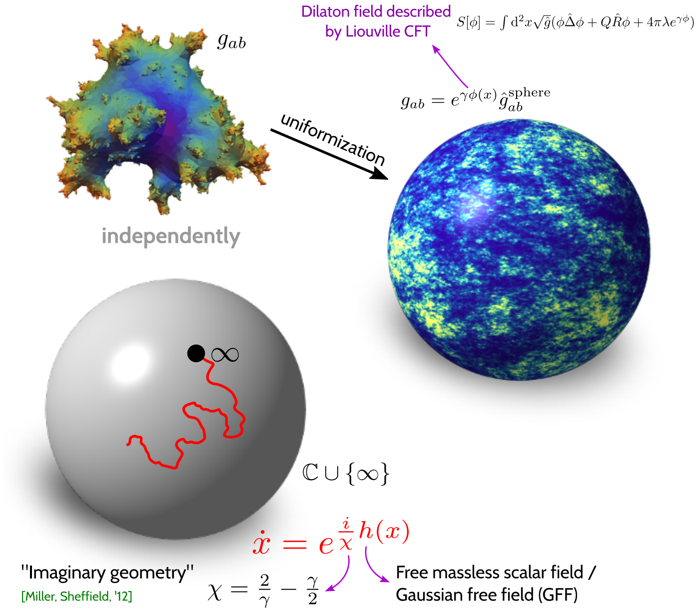
$$g_{ab} = e^{\gamma\phi(x)}\hat{g}_{ab}^{\text{sphere}}$$



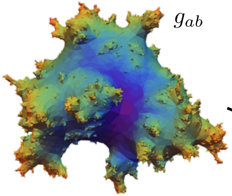
# Why does this work?



# Why does this work?



# Why does this work?



$g_{ab}$

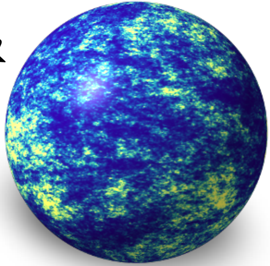
independently

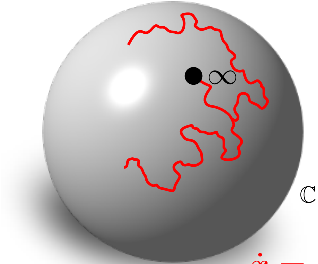
uniformization

Dilaton field described by Liouville CFT

$S[\phi] = \int \mathrm{d}^2x \sqrt{\hat{g}} (\phi \hat{\Delta} \phi + Q\hat{R}\phi + 4\pi\lambda e^{\gamma\phi})$

$g_{ab} = e^{\gamma\phi(x)} \hat{g}_{ab}^{\text{sphere}}$






$\mathbb{C} \cup \{\infty\}$

$\dot{x} = e^{\frac{i}{\chi} h(x)}$

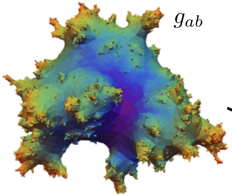
"Imaginary geometry"

[Miller, Sheffield, '12]

$\chi = \frac{2}{\gamma} - \frac{\gamma}{2}$

Free massless scalar field / Gaussian free field (GFF)

# Why does this work?



$g_{ab}$

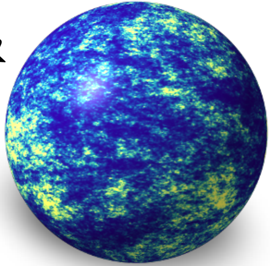
independently

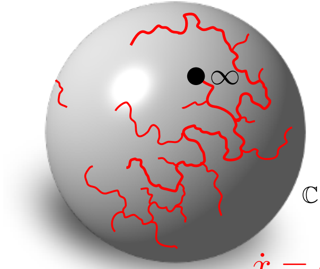
uniformization

Dilaton field described by Liouville CFT

$S[\phi] = \int \mathrm{d}^2 x \sqrt{\hat{g}} (\phi \hat{\Delta} \phi + Q \hat{R} \phi + 4\pi\lambda e^{\gamma\phi})$

$g_{ab} = e^{\gamma\phi(x)} \hat{g}_{ab}^{\text{sphere}}$







$\mathbb{C} \cup \{\infty\}$

$\dot{x} = e^{\frac{i}{\chi} h(x)}$

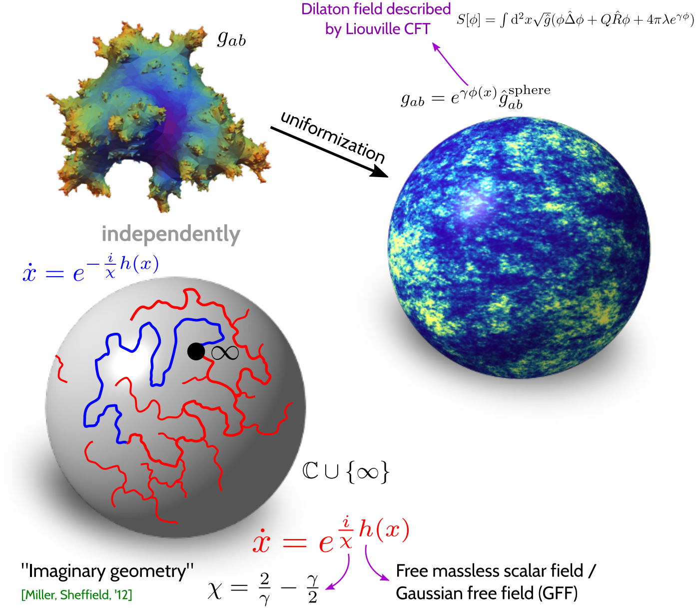
"Imaginary geometry"

[Miller, Sheffield, '12]

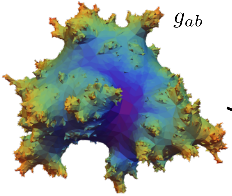
$\chi = \frac{2}{\gamma} - \frac{\gamma}{2}$

Free massless scalar field / Gaussian free field (GFF)

# Why does this work?



# Why does this work?



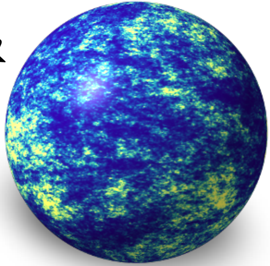
$g_{ab}$

Dilaton field described by Liouville CFT

$S[\phi] = \int \mathrm{d}^2x \sqrt{\hat{g}}(\phi\hat{\Delta}\phi + Q\hat{R}\phi + 4\pi\lambda e^{\gamma\phi})$

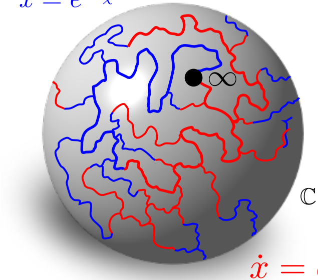
$g_{ab} = e^{\gamma\phi(x)}\,\hat{g}_{ab}^{\text{sphere}}$

uniformization



independently

$\dot{x} = e^{-\frac{i}{\chi}h(x)}$




$\mathbb{C} \cup \{\infty\}$

$\dot{x} = e^{\frac{i}{\chi}h(x)}$

"Imaginary geometry"

[Miller, Sheffield, '12]

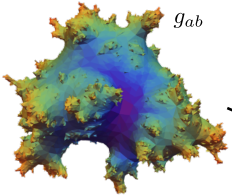
$\chi = \frac{2}{\gamma} - \frac{\gamma}{2}$

Free massless scalar field / Gaussian free field (GFF)

# Why does this work?

Dilaton field described by Liouville CFT

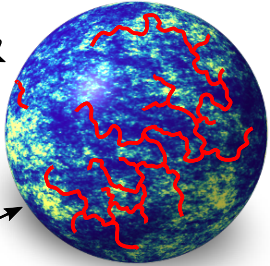
$S[\phi] = \int \mathrm{d}^2x\sqrt{\hat{g}}(\phi\hat{\Delta}\phi + Q\hat{R}\phi + 4\pi\lambda e^{\gamma\phi})$



$g_{ab}$

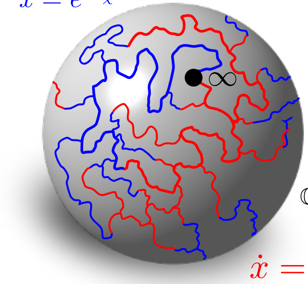
uniformization

$g_{ab} = e^{\gamma\phi(x)}\,\hat{g}_{ab}^{\text{sphere}}$



independently

$\dot{x} = e^{-\frac{i}{\chi}h(x)}$



$\infty$

$\mathbb{C}\cup\{\infty\}$

measure branches with metric $g_{ab}$

$\dot{x} = e^{\frac{i}{\chi}h(x)}$

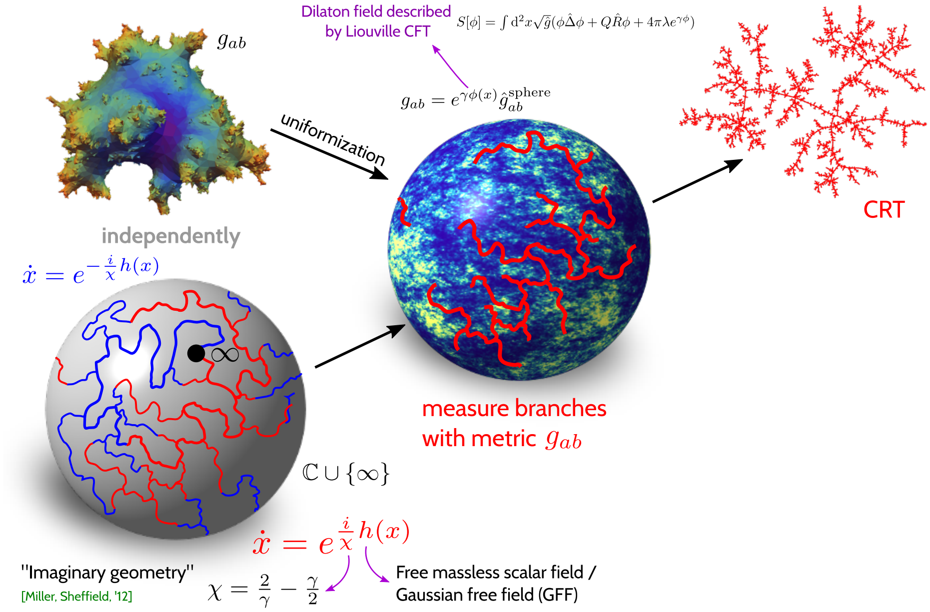
"Imaginary geometry"

[Miller, Sheffield, '12]

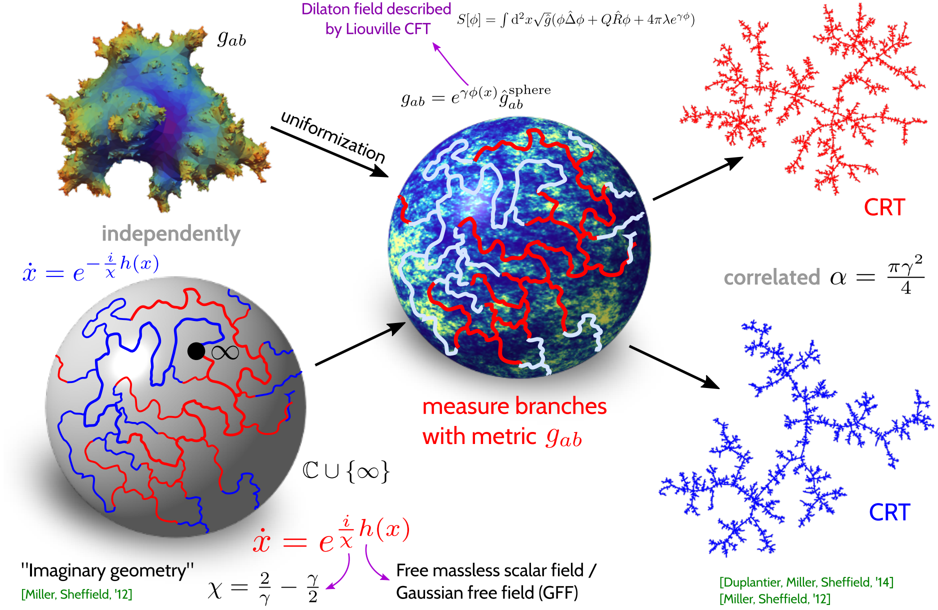
$\chi = \frac{2}{\gamma} - \frac{\gamma}{2}$

Free massless scalar field / Gaussian free field (GFF)

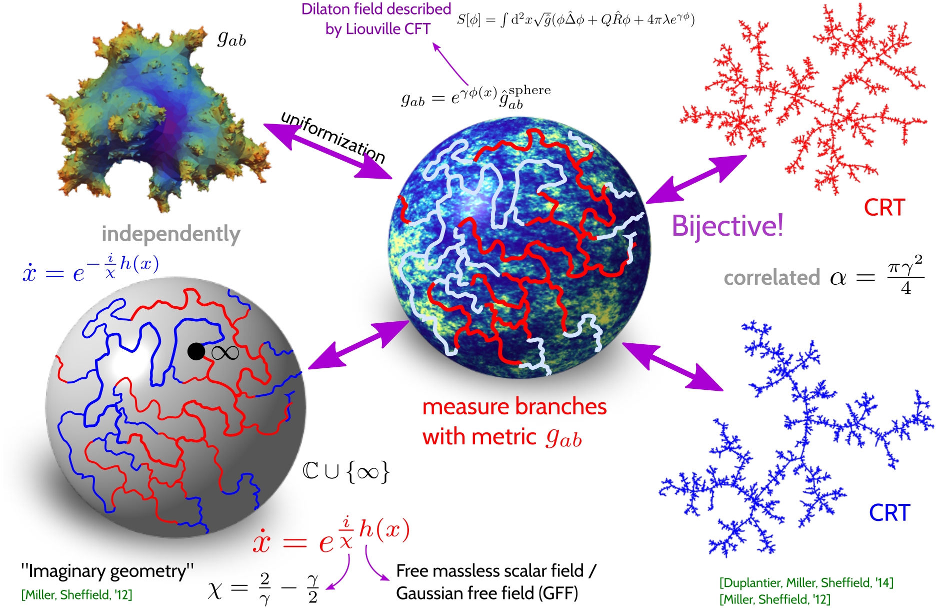
# Why does this work?



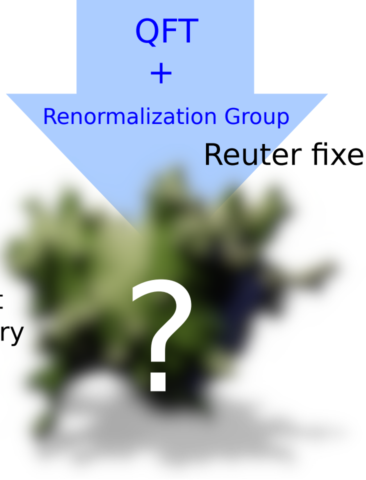
# Why does this work?



# Why does this work?



# in 3D
(or higher)



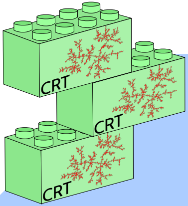
# in 3D
(or higher)

QFT
+
Renormalization Group

Reuter fixed point?

?

scale-invariant
random geometry




with

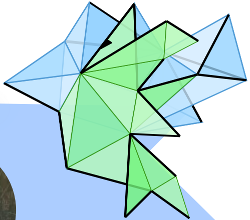
Alicia Castro

[arxiv:2206.xxxxx]

Assembly approach

with

Luca Lionni



[arxiv:2203.16105]

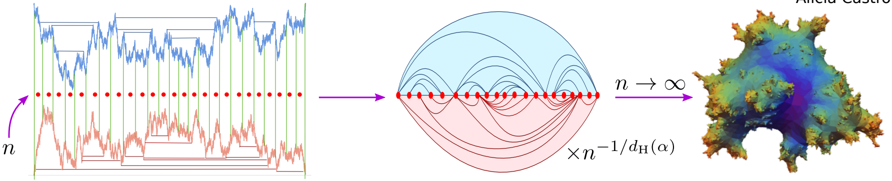
Lattice approach

Pending required mathematics to study generalization of the assembly: develop a numerical toolbox.

with

Alicia Castro

First benchmark in 2D quantum gravity!




Mated-CRT map

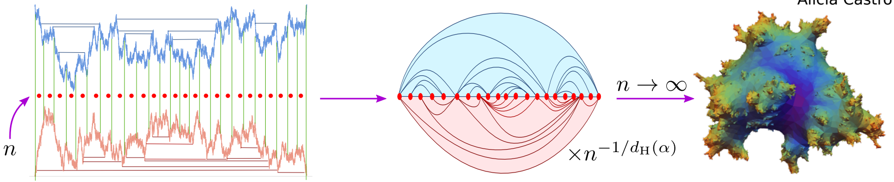
[Gwynne, Miller, Sheffield, '17]

Pending required mathematics to study generalization of the assembly: develop a numerical toolbox.

First benchmark in 2D quantum gravity!

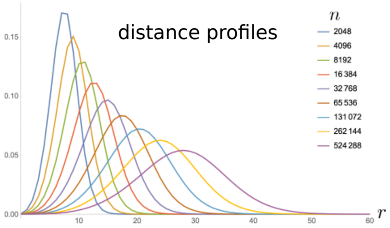
with

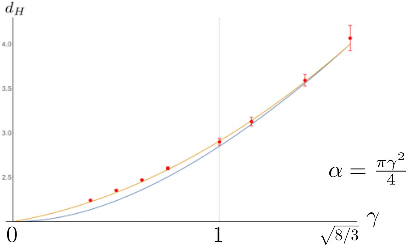
Alicia Castro




Mated-CRT map
[Gwynne, Miller, Sheffield, '17]

Hausdorff dimension measurements consistent with previous estimates:

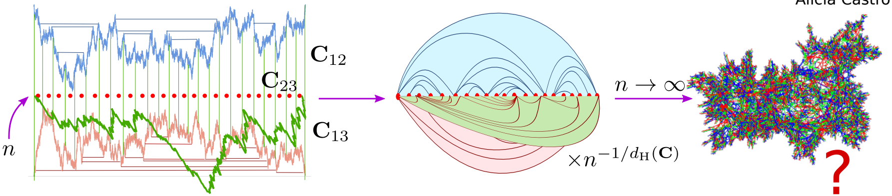


Pending required mathematics to study generalization of the assembly: develop a numerical toolbox.

with

Alicia Castro

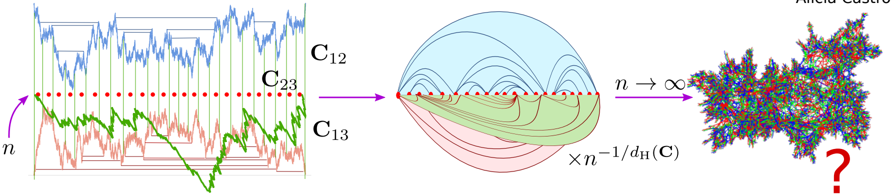



Adding a 3rd CRT: 3-dimensional phase space and can be estimated.

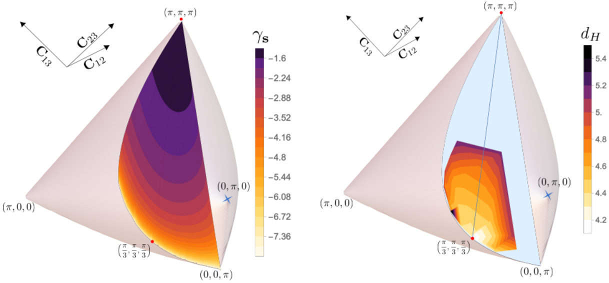
Pending required mathematics to study generalization of the assembly: develop a numerical toolbox.

with

Alicia Castro




Adding a 3rd CRT: 3-dimensional phase space and can be estimated.



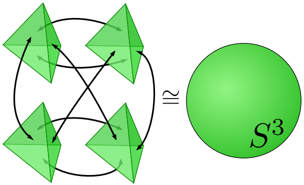

New scale-invariant random geometry, but does it have 3-manifold topology?

# Universality from 3d discrete geometries?

- Lattice approach: continuum limit of random discrete geometries at criticality.

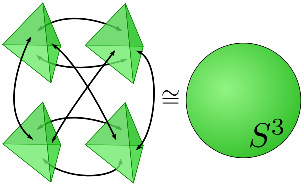
# Universality from 3d discrete geometries?

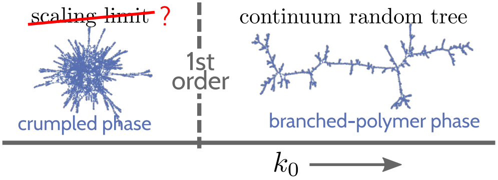
- Lattice approach: continuum limit of random discrete geometries at criticality.
- 3D Dynamical Triangulations: random triangulations of $S^3$ sampled proportional to $e^{k_0 \# \text{vertices}}$ [Ambjorn, Durhuus, Jonsson, Sasakura, Godrfrey, Gross, Varsted, Boulatov, Agishtein, Migdal, . . . , '91]



# Universality from 3d discrete geometries?

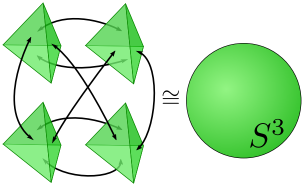
- Lattice approach: continuum limit of random discrete geometries at criticality.
- 3D Dynamical Triangulations: random triangulations of $S^3$ sampled proportional to $e^{k_0 \#\text{vertices}}$ [Ambjorn, Durhuus, Jonsson, Sasakura, Godrfrey, Gross, Varsted, Boulatov, Agishtein, Migdal, . . . , '91]
- Limited success:
  1. lack of mathematical control (no good topological invariants / missing exponential bound);
  2. 1d phase diagram numerically shows no new critical phenomena.

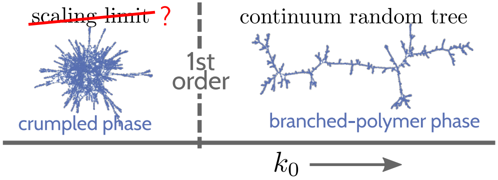


# Universality from 3d discrete geometries?

- Lattice approach: continuum limit of random discrete geometries at criticality.
- 3D Dynamical Triangulations: random triangulations of $S^3$ sampled proportional to $e^{k_0 \#\text{vertices}}$ [Ambjorn, Durhuus, Jonsson, Sasakura, Godrfrey, Gross, Varsted, Boulatov, Agishtein, Migdal, ..., '91]
- Limited success:
  1. lack of mathematical control (no good topological invariants / missing exponential bound);
  2. 1d phase diagram numerically shows no new critical phenomena.

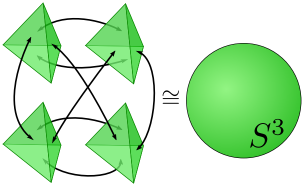


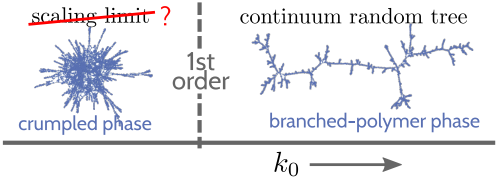




- Option: introduce a causal structure → Causal Dynamical Triangulations (CDT) [Loll, Ambjorn, Jurkiewicz,...]

# Universality from 3d discrete geometries?

- Lattice approach: continuum limit of random discrete geometries at criticality.
- 3D Dynamical Triangulations: random triangulations of $S^3$ sampled proportional to $e^{k_0 \# \text{vertices}}$ [Ambjorn, Durhuus, Jonsson, Sasakura, Godrfrey, Gross, Varsted, Boulatov, Agishtein, Migdal, . . . , '91]
- Limited success:
  1. lack of mathematical control (no good topological invariants / missing exponential bound);
  2. 1d phase diagram numerically shows no new critical phenomena.



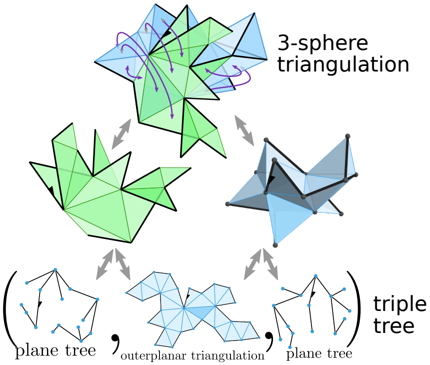




- Option: introduce a causal structure $\rightarrow$ Causal Dynamical Triangulations (CDT) [Loll, Ambjorn, Jurkiewicz,. . . ]
- Other Idea: identify tree structures within these geometries to facilitate analytic methods and enhance phase diagram.

# An explicit model. [TB, Lionni, '22]

- There exists a bijection

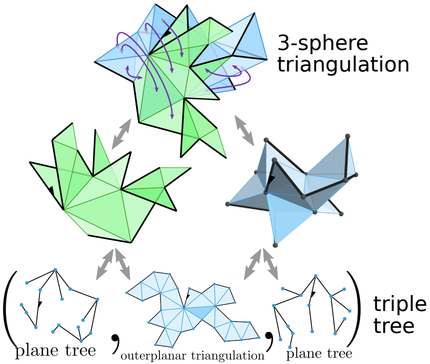
$$\left\{ \begin{array}{c} \text{subclass of triangulated 3-spheres} \\ \text{“tree-avoiding locally constructible”} \end{array} \right\} \leftrightarrow \left\{ \text{triple trees} \right\}$$



# An explicit model. [TB, Lionni, '22]

- There exists a bijection

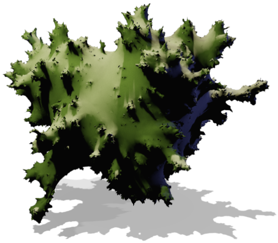
$$\left\{ \begin{array}{c} \text{subclass of triangulated 3-spheres} \\ \text{"tree-avoiding locally constructible"} \end{array} \right\} \leftrightarrow \left\{ \text{triple trees} \right\}$$




- Combinatorial enumeration still open, but shows promising numerical properties.

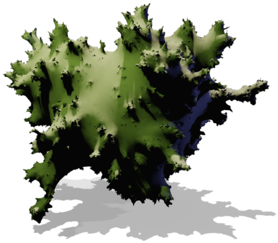
# Conclusions

- If gravity is asymptotically safe, microscopic spacetime geometry is nothing like that of GR.
- Need to prepare ourselves with all the mathematical and numerical tools on offer/in development in Random Geometry.
- (Almost) all known universality classes of random geometry can be assembled from the Continuum Random Tree (CRT).
- We may be seeing the first universality beyond trees and surfaces, but 3D geometry without regularity is a wilderness. . . .



# Conclusions

- If gravity is asymptotically safe, microscopic spacetime geometry is nothing like that of GR.
- Need to prepare ourselves with all the mathematical and numerical tools on offer/in development in Random Geometry.
- (Almost) all known universality classes of random geometry can be assembled from the Continuum Random Tree (CRT).
- We may be seeing the first universality beyond trees and surfaces, but 3D geometry without regularity is a wilderness. . . .



Thanks. Questions?